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Modelling Data – Better Approaches

How to get useful information?

Adrian R. Rennie



Monolayers – Simple Interpretation

Define $g_s(Q_z)$ in terms of measured reflectivity and $R_F(Q_z)$ (the Fresnel reflectivity for perfectly sharp interface):

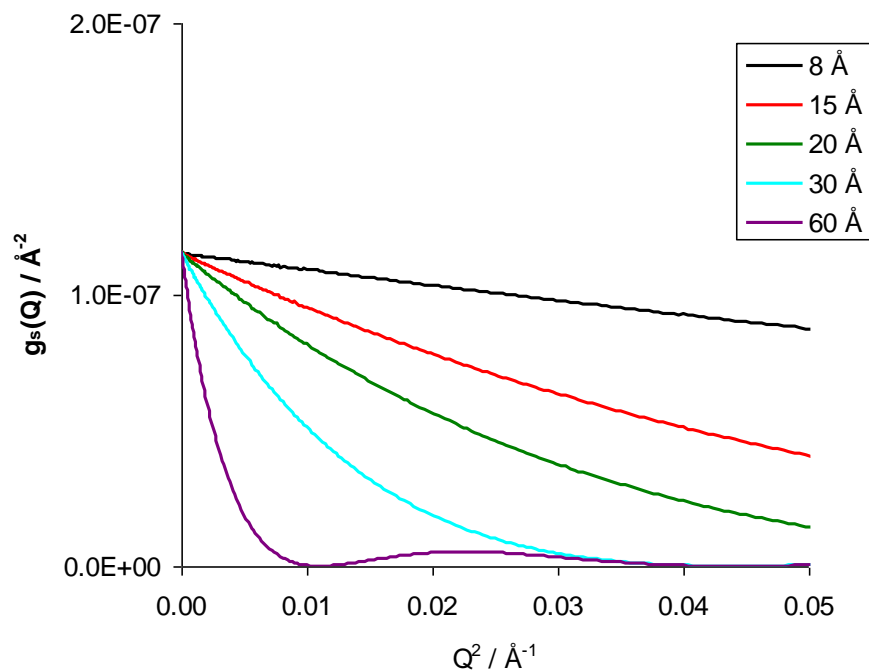
$$g_s(Q) = Q^2 (R - R_F) / (1 - R)$$

$$\ln g_s(Q) \approx -\ell^2 Q^2 / 12$$

$$\text{Roughly } \ln(Q^2 R) \approx -\ell^2 Q^2 / 12$$

Contrast match of two bulk phases

$$R_F(Q) = 0$$

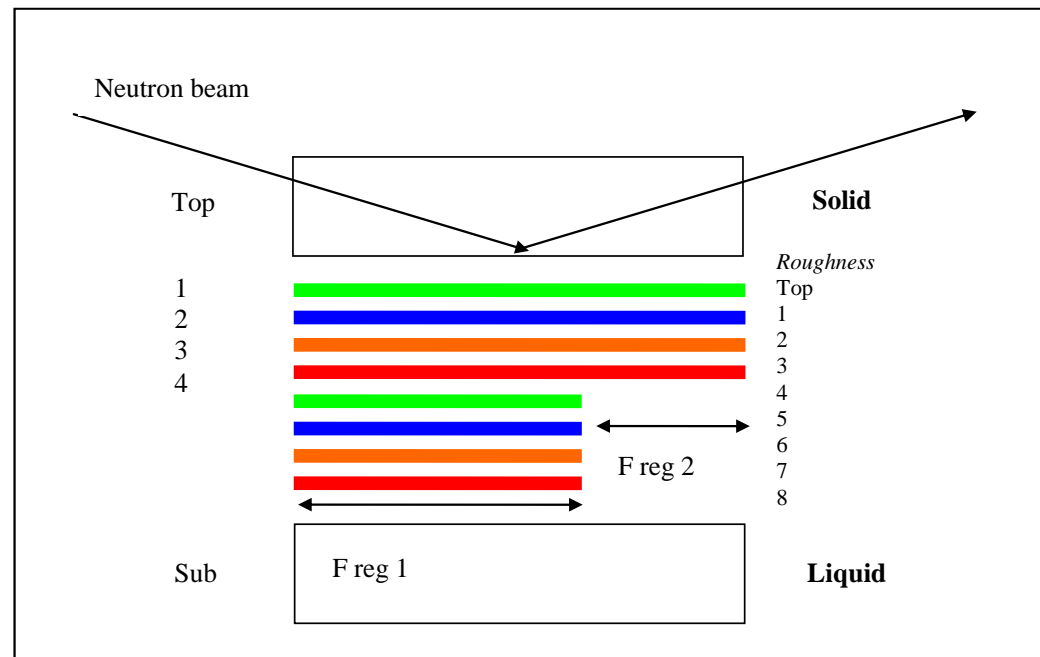




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Real Interfaces are not just layers

Slab models are easy to calculate but people are not very interested in just thickness and scattering length density





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Surface Excess and Area per Molecule

Volume per molecule:

Scattering length: b_m

Scattering length density:

$$\rho = b_m / V_m$$

Thickness of layer:

Scattering length density

Area per molecule:

$$V_m = t A_m$$

Scattering length density:

$$\rho = (b_m / V_m) = b_m / (t A_m)$$

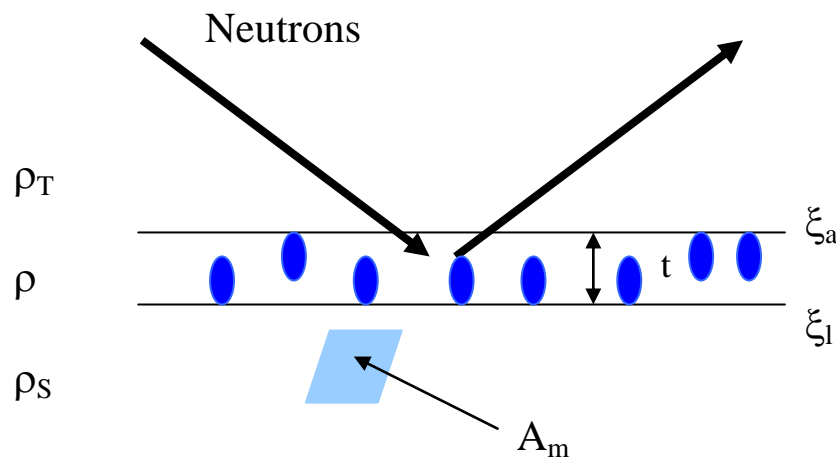
Area per molecule: $A_m = b_m / t \rho$

V_m

t

ρ

A_m





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Adsorption of Surfactant

Surface active molecules

Amphiphilic

Bind to surface – how?

What are properties?

Hexadecyl trimethyl
ammonium bromide



Tail

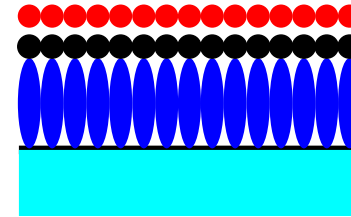
Head



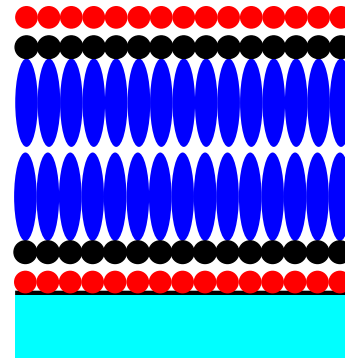
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Some Possible Structures

- Monolayer



- Bilayer





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Cationic Surfactant

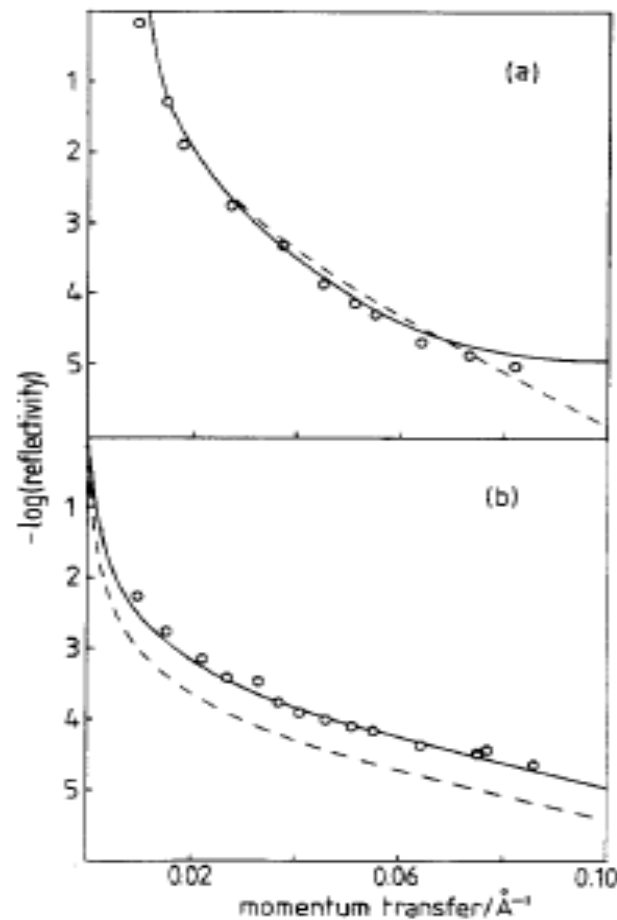
CTAB at 27° C on
amorphous SiO₂

(a) D₂O (b) cmSiO₂
at 6 × 10⁻⁴ M

Models

Solid line – Bilayer

Dashed line - Monolayer





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Cationic Surfactant

- **CTAB** 27 C on SiO₂
- Label heads & tails

Head 6 +/- 2 Å

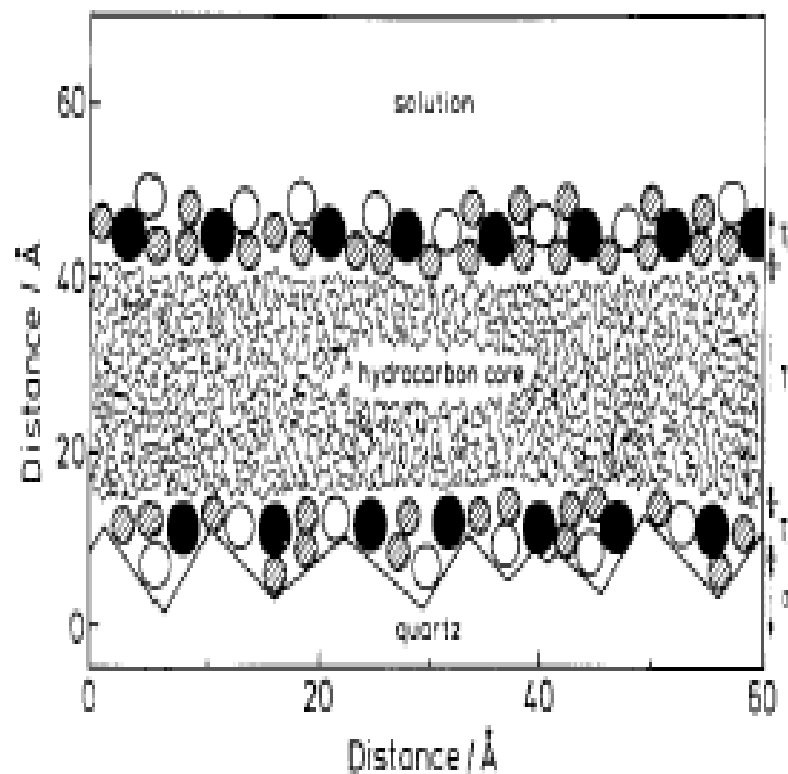
Tail 28 +/- 4 Å

Roughness ~ 8 Å

Fractional Coverage

35% at 3×10^{-4} M

80% at 6×10^{-4} M



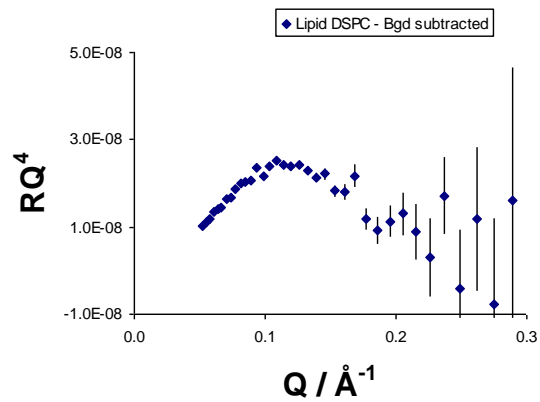
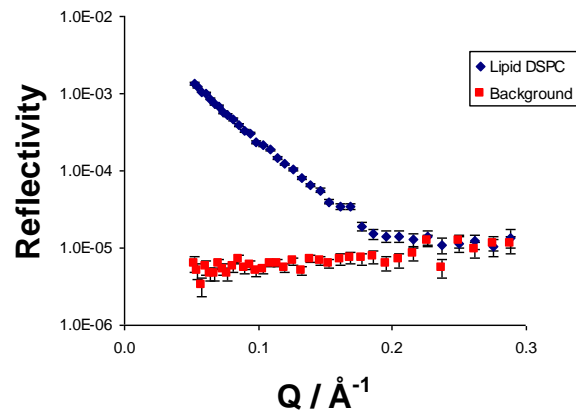
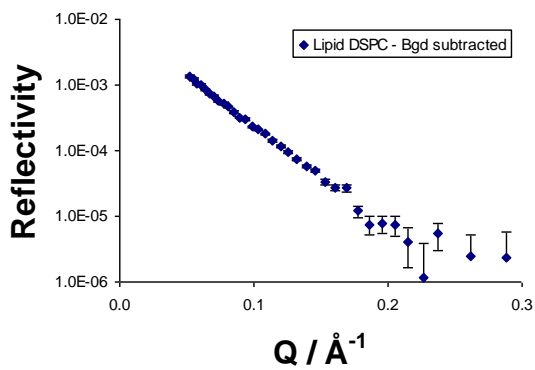
Langmuir **6**, 1031-1034 (1990).
J. Colloid Interf. Sci. **162**, 304-310 (1994).



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Plotting Data

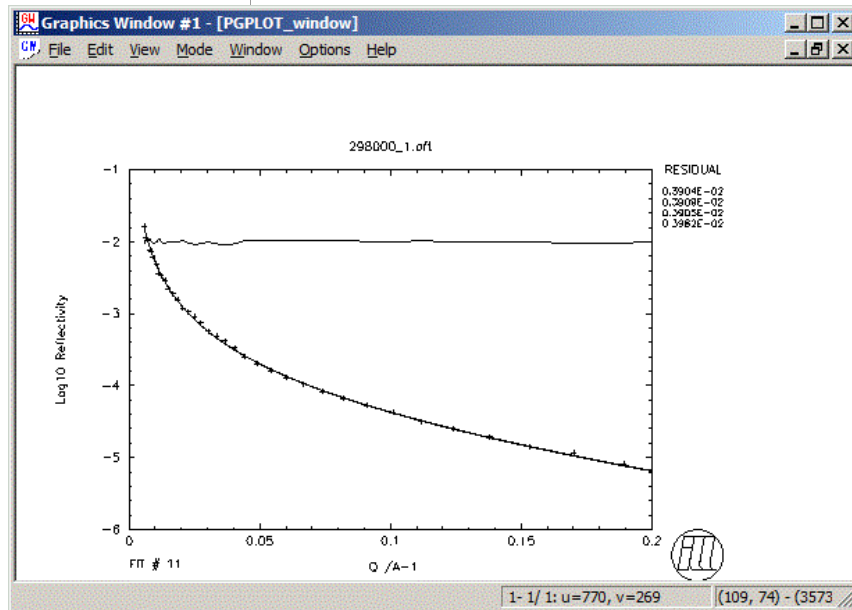
Different representation
is helpful



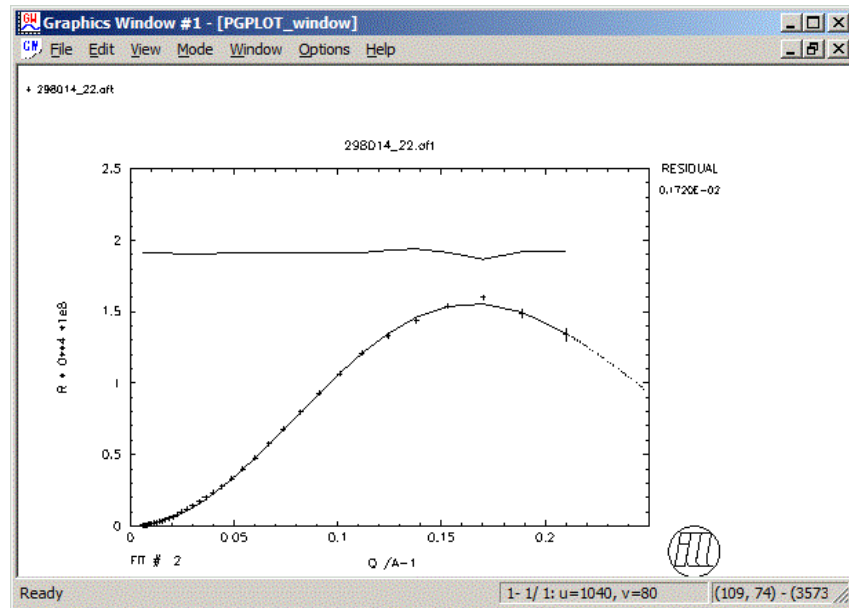


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How to Look at Data?



Log₁₀ R vs Q

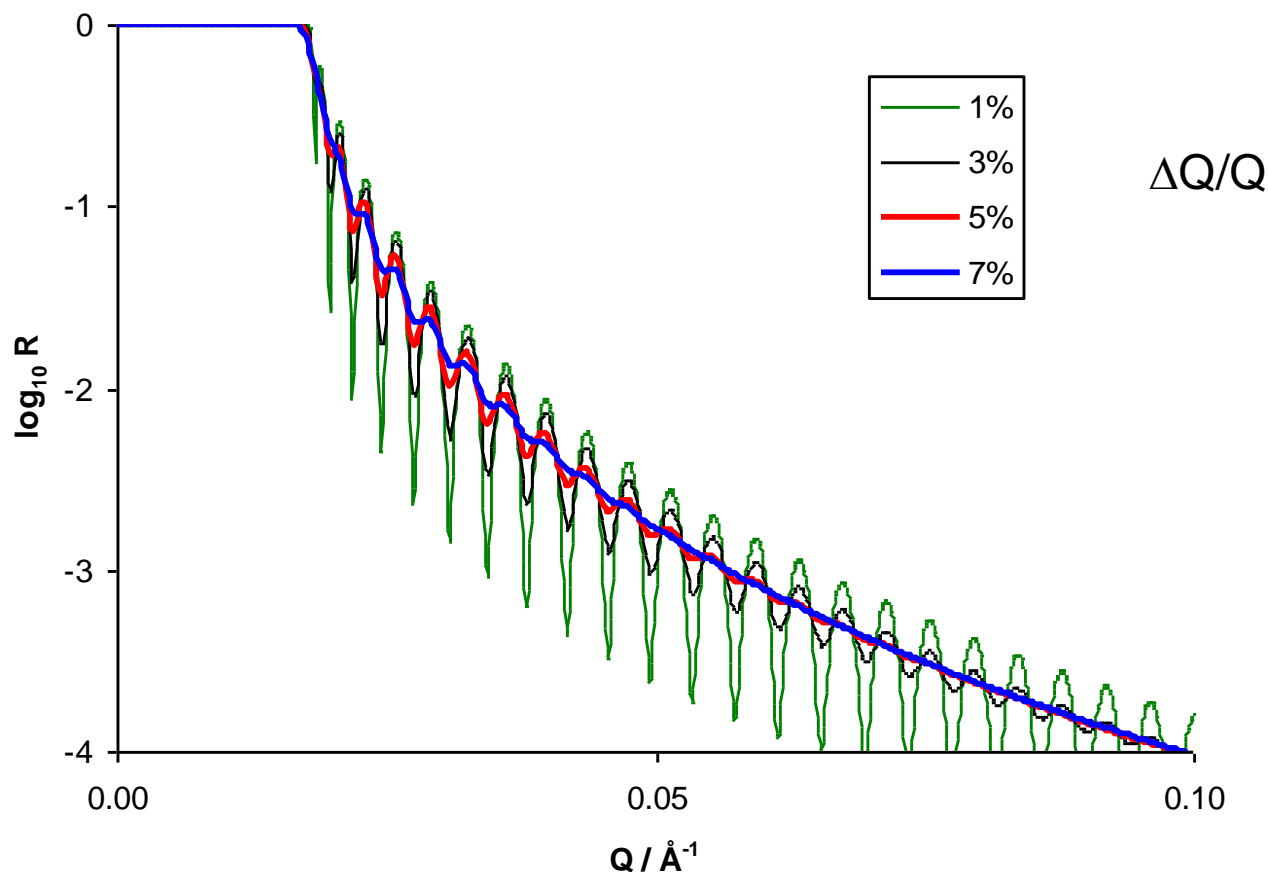


RQ⁴ vs Q



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Effects of Resolution

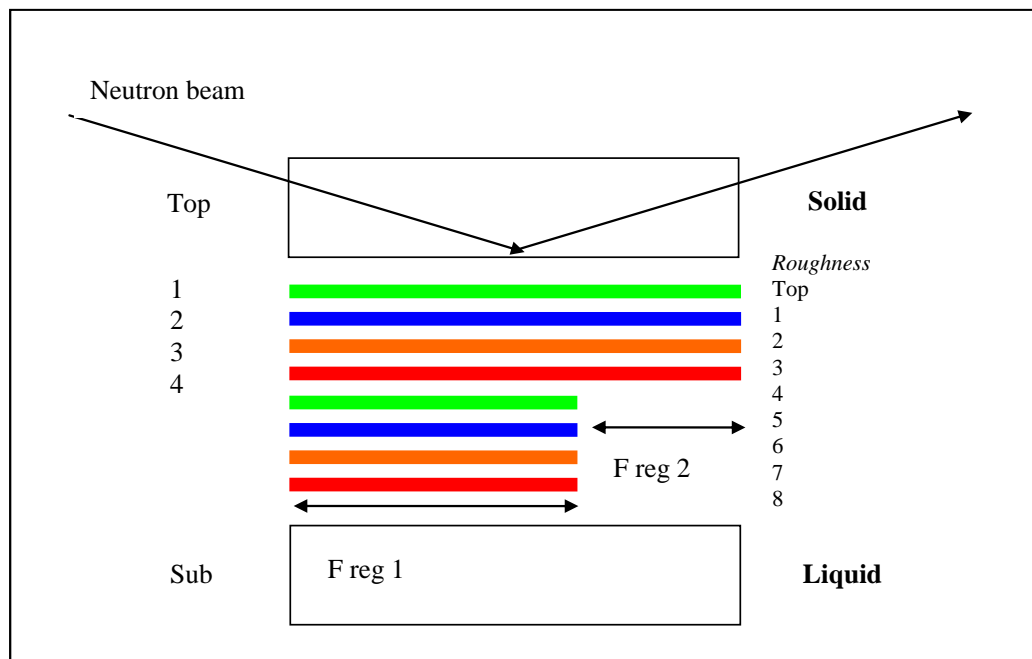


Silicon substrate: film thickness 1500 \AA
scattering length density $6.3 \times 10^{-6} \text{\AA}^{-2}$



Non-Uniform Surfaces

If you have patches of different layers at an interface do you average the density or average the reflectivity?



What is the coherence length of a neutron?



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Describing Polymers

- Interdiffusion – is this roughness?
- Brushes – parabolic density profile
(E. P. K. Currie et al *Physica B*, **283** 17 – 21)
- Other scaling laws e.g. O. Guiselin *J. Phys.* **50**, 3407-3425 (1989).

We expect smooth profiles!



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Thermoresponsive polymer brush

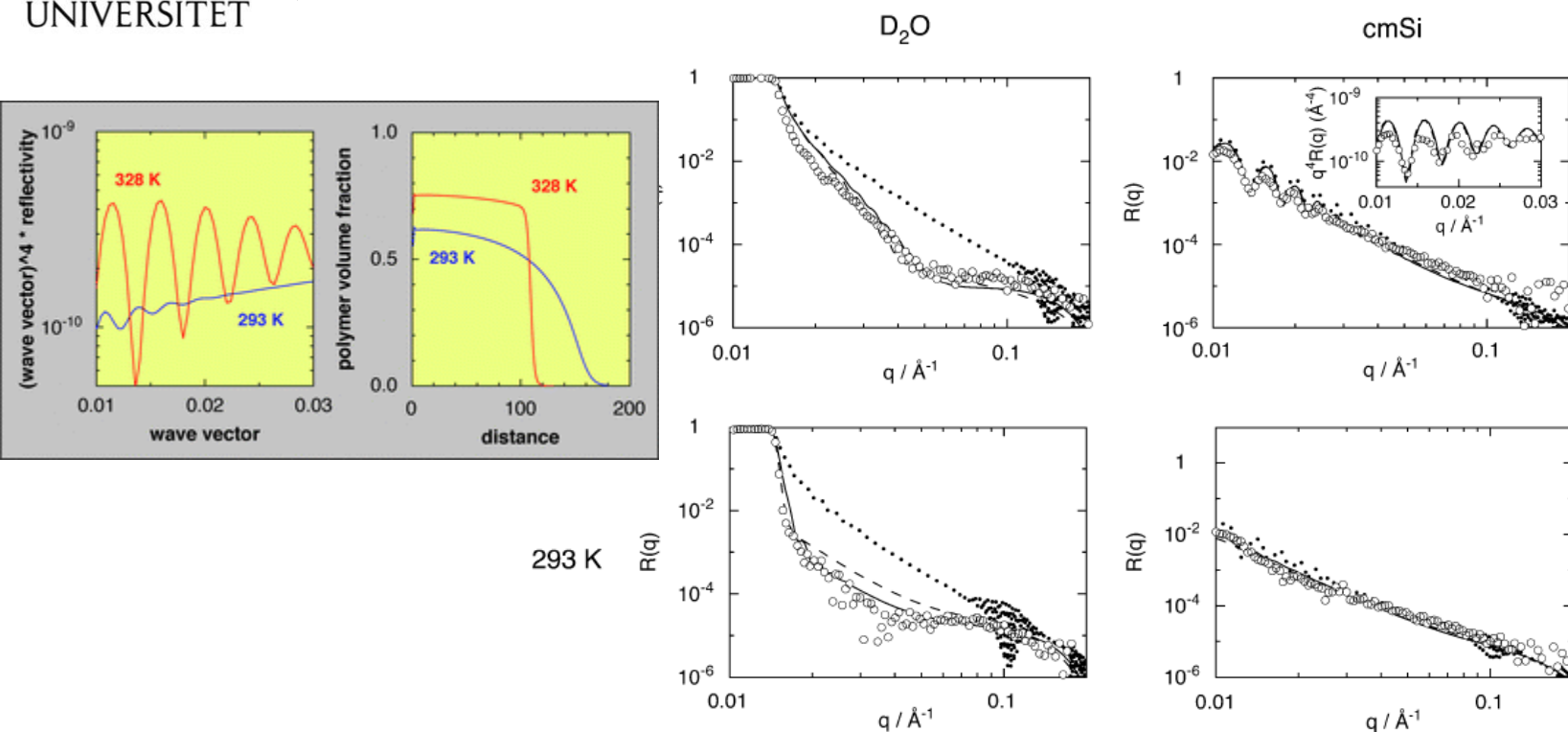


Fig. 6 Experimental reflectivity profiles obtained at ILL (circles) and fitted reflectivity profiles using a polymer layer model (dashed curves) and a lattice mean-field theory (solid curves) for polymers grafted on a Si/SiO₂/initiator surface at 328 K (top) and 293 K (bottom) in D₂O (left) and cmSi (right). Reflectivity profiles using a polymer layer model with zero roughness are also shown (dotted curves). The top right panel contains an inset displaying $q^4 R(q)$ versus q for small q .

J. Zhang, et al., *Soft Matter*, 4, 500–509 (2008).



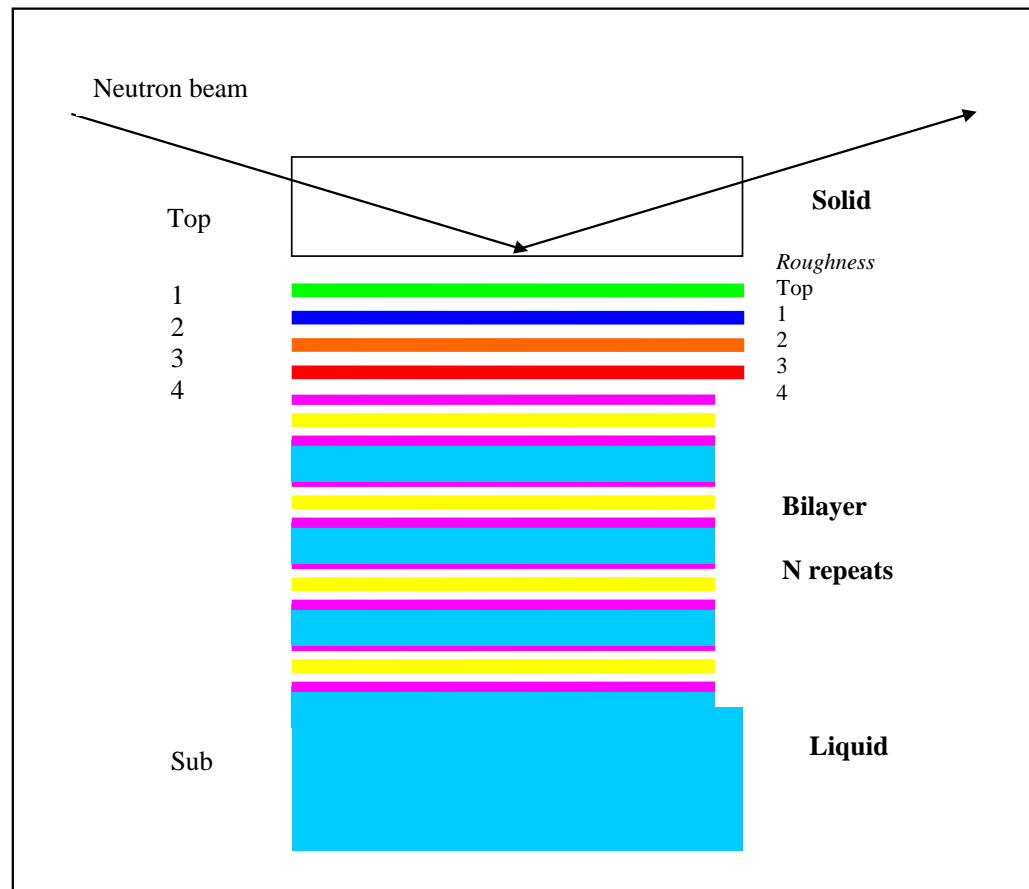
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Repeating Layers

A one dimensional
crystal

Bragg's law

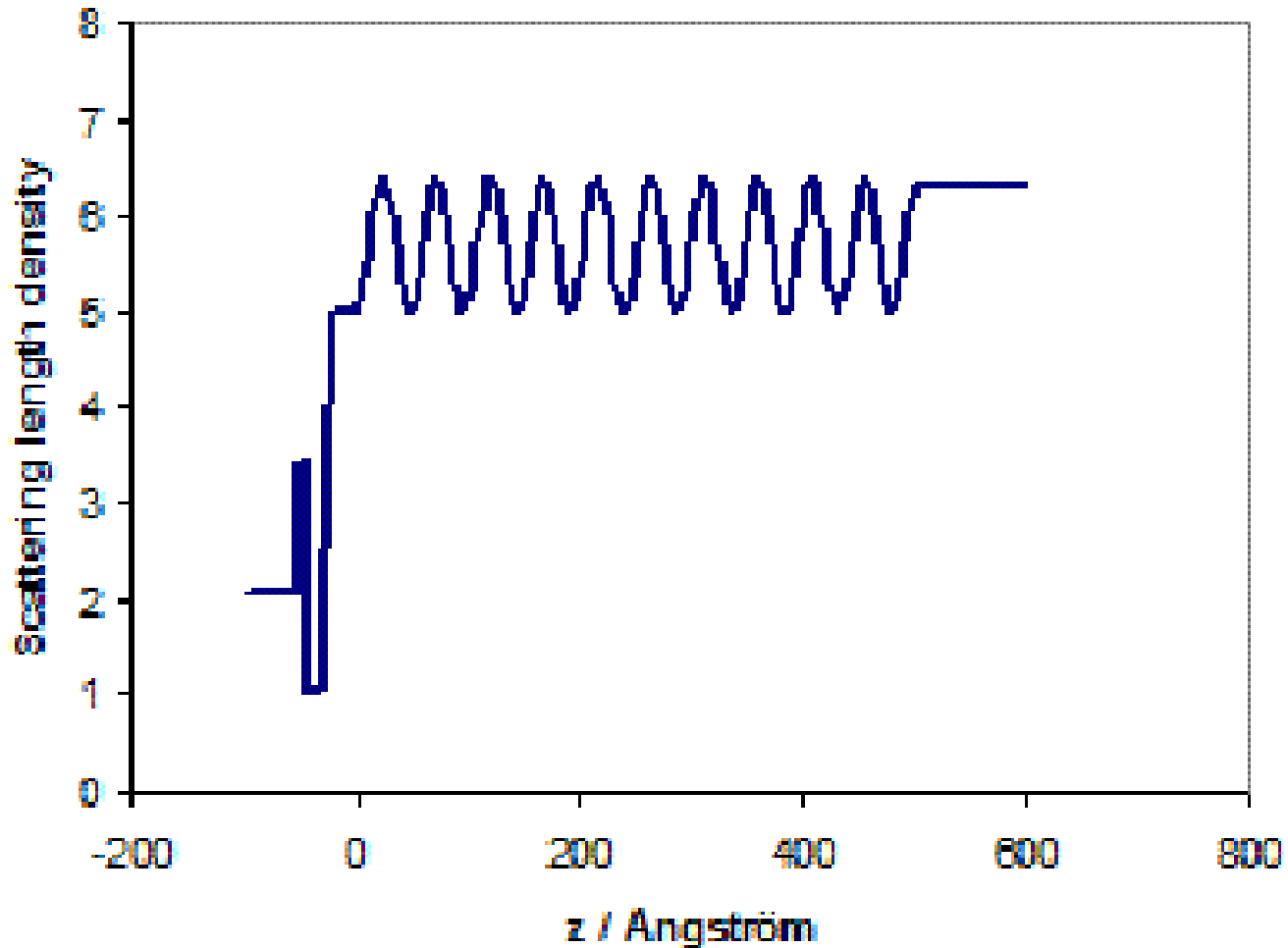
Intensity of peaks may
Depend on size and
disorder





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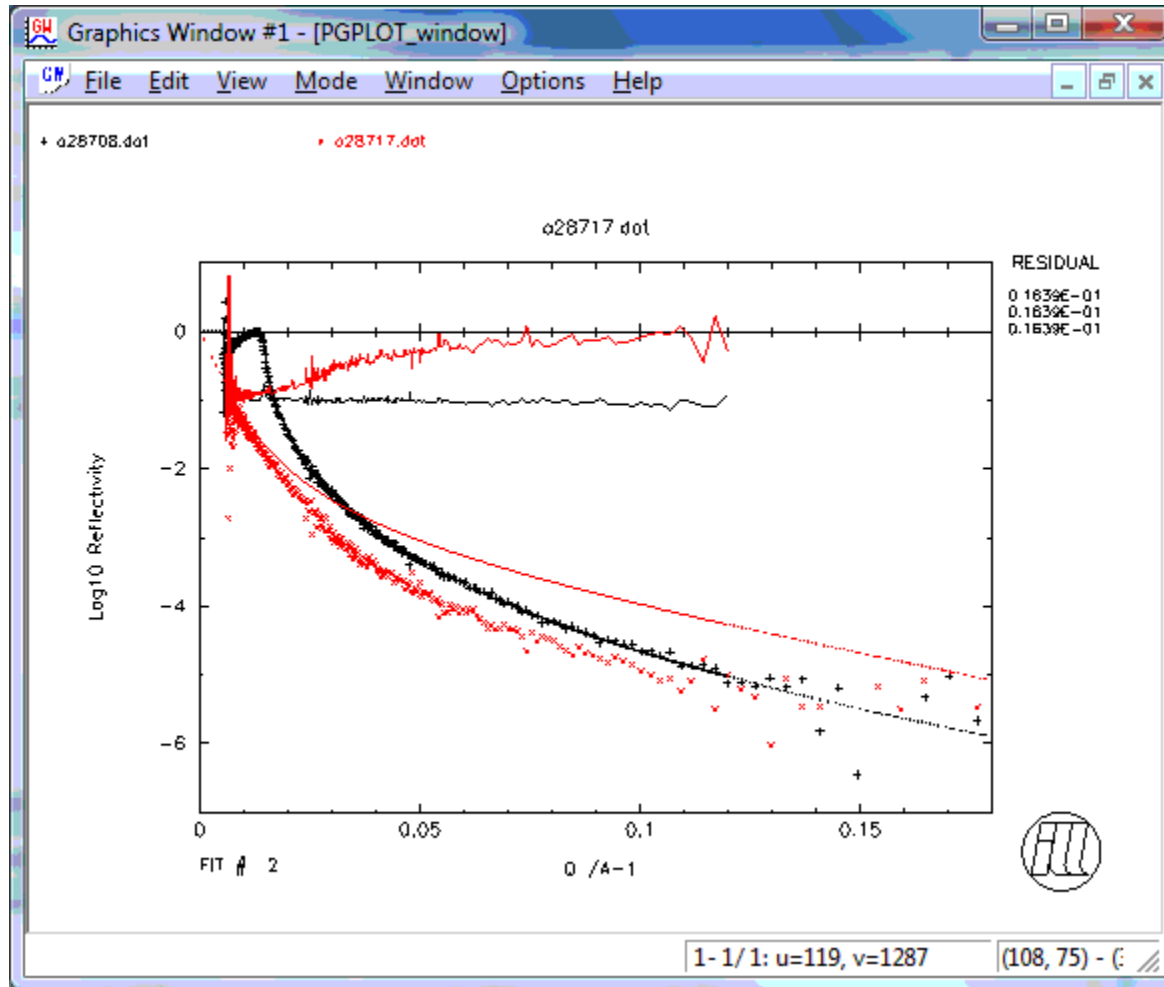
Calculate reflectivity for a profile





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Using Multiple Contrasts



Simultaneous fits for multiple data sets



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More Ideas

www.reflectometry.net



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Off-specular Scattering, GISANS, Near-surface SANS

Adrian R. Rennie



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SANS and GISANS

- Transmission geometry SANS is usually a simpler experiment
- In principle, calculations are identical

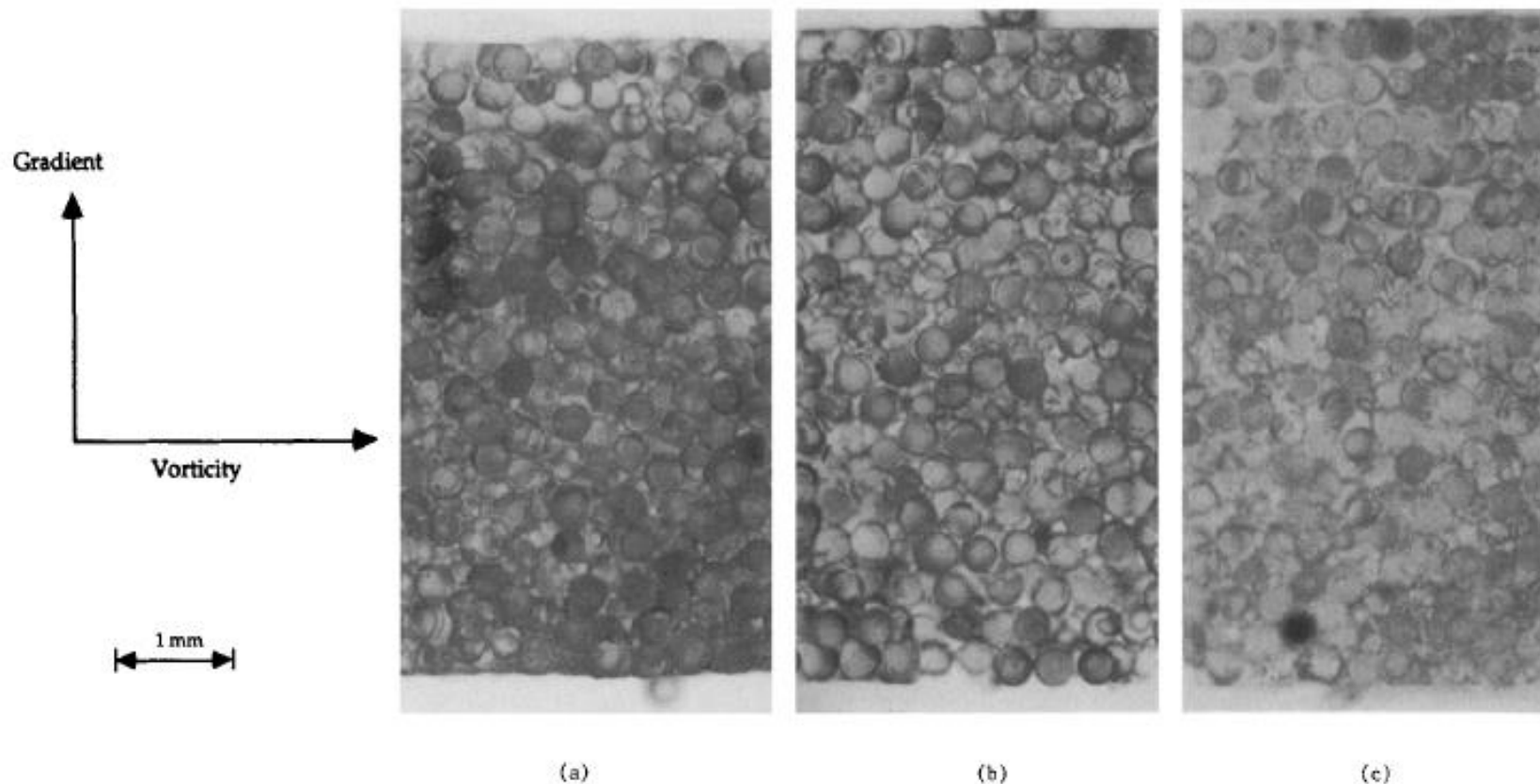
BUT

Geometry and Multiple Scattering are important



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Interfaces are 3-dimensional



Understanding rheology – shear flow

Brown et al. *Progress in Colloid and Polymer Science* **98**, (1995) 99-102.

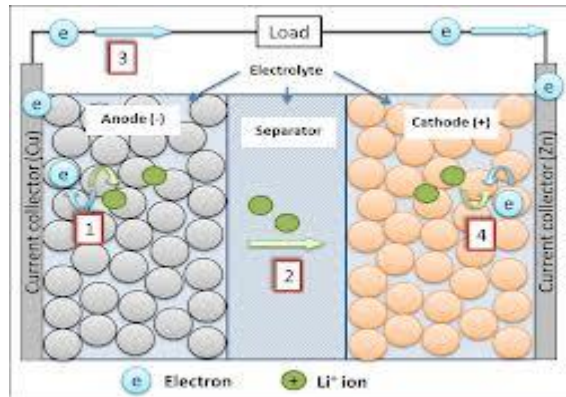


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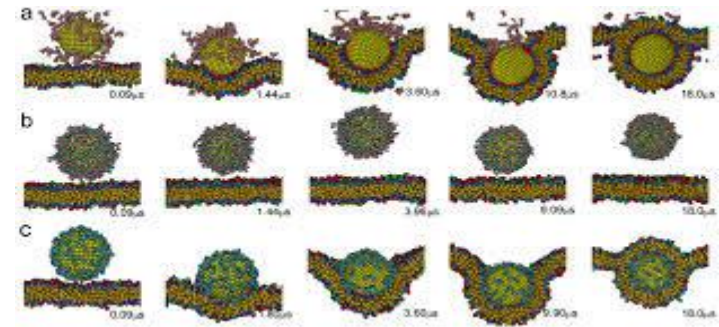
Interfaces - Where things happen?



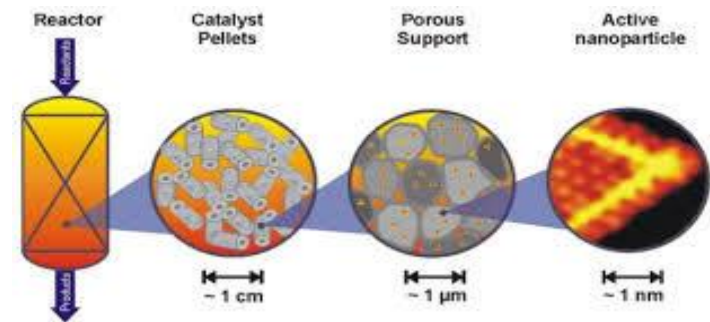
Lubrication – SKF bearing



Electrode – Battery Oxford



Nanotoxicology - *Nature*



Catalysts



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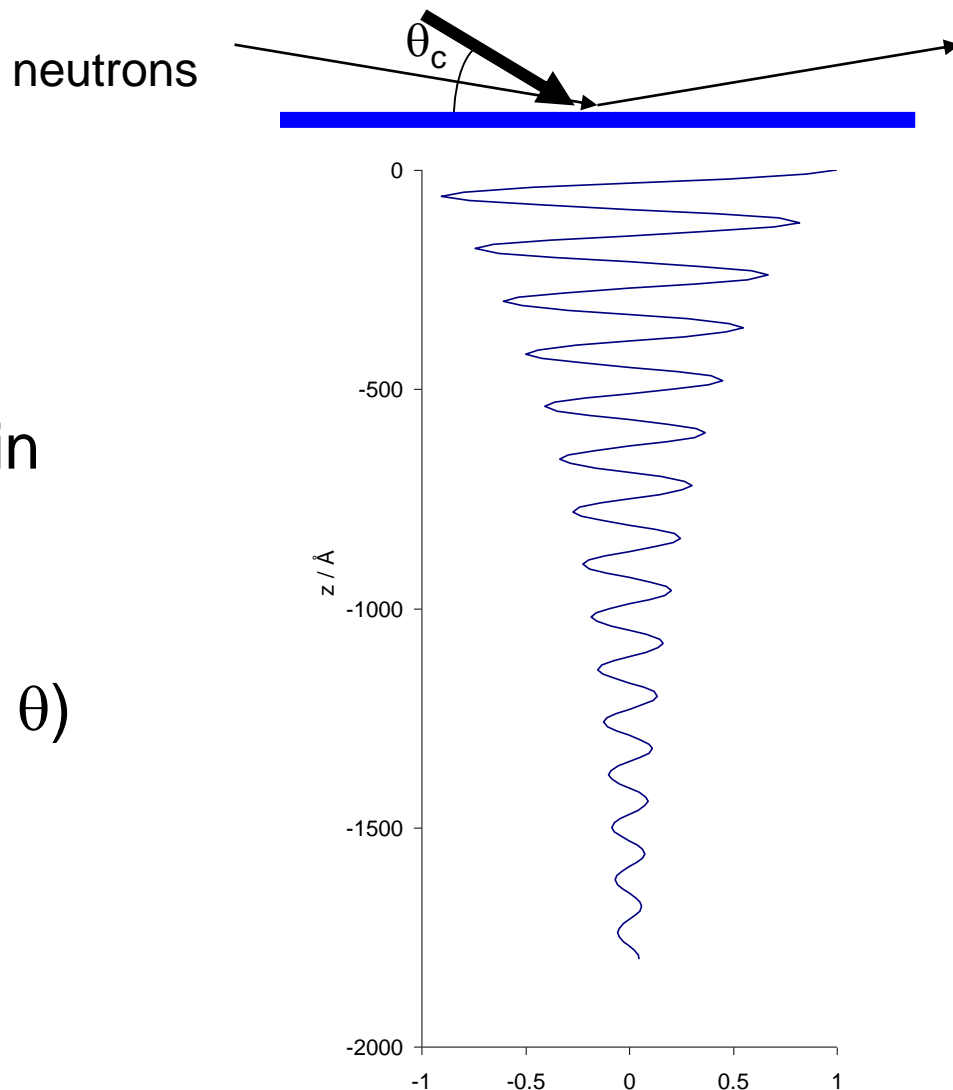
Evanescent Wave

Below k_c no travelling wave enters the sample

Amplitude decays with depth in sample

Decay length depends on $(\theta_c - \theta)$

Evanescent wave can cause scattering





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Looking at Materials



Anneli Salo - Own work, CC BY-SA 3.0,
<https://commons.wikimedia.org/w/index.php?curid=6746303>



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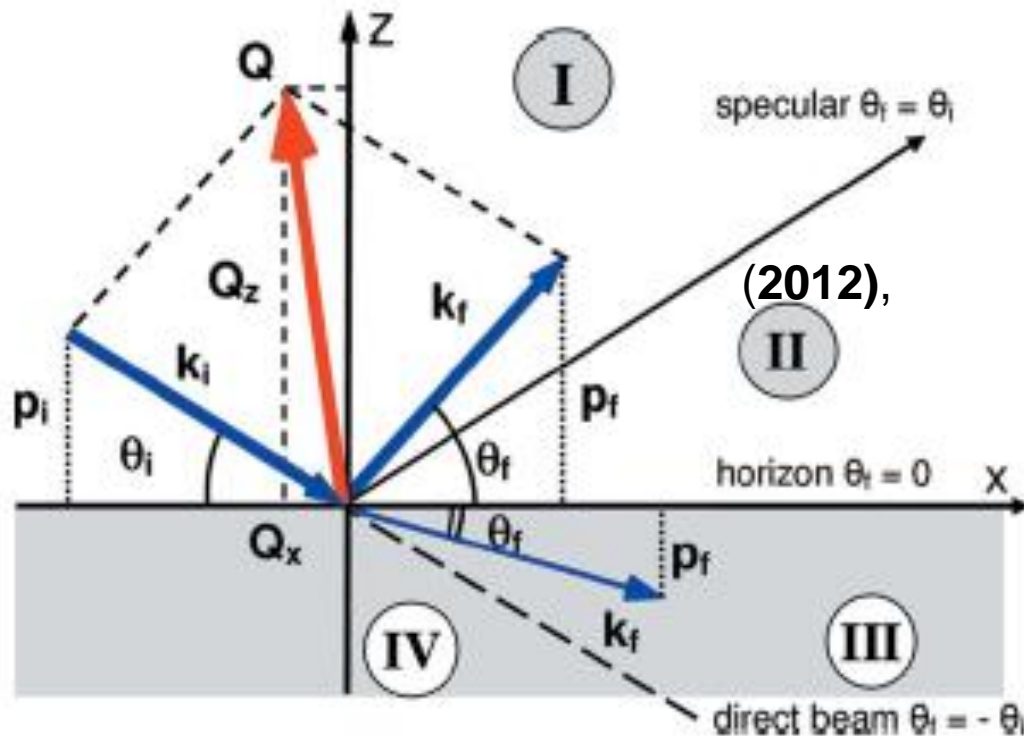
Looking at Materials



Anneli Salo - Own work, CC BY-SA 3.0,
<https://commons.wikimedia.org/w/index.php?curid=6746303>



Off-specular & Reflection



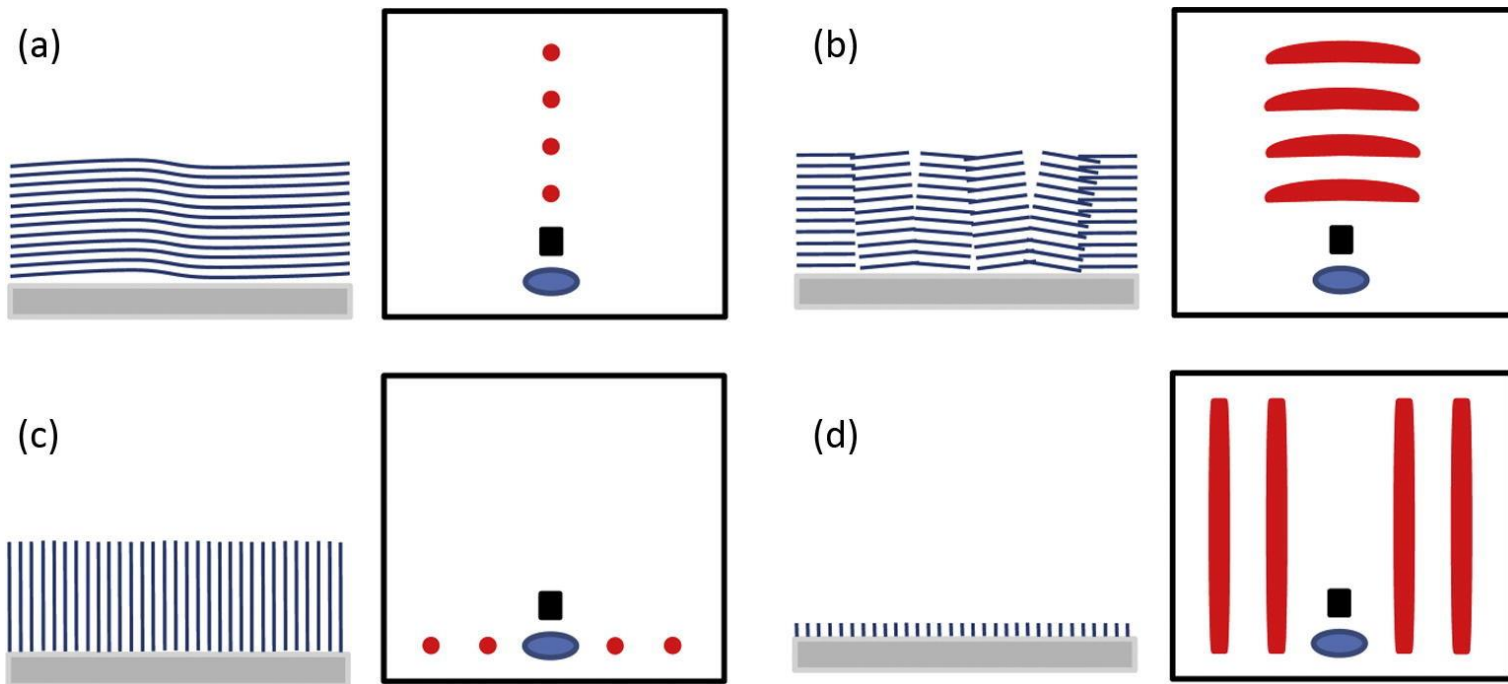
$$Q_z \approx (2\pi/\lambda) (\theta_i + \theta_f)$$

$$Q_x \approx (2\pi/\lambda) (\theta_i + \theta_f) (\theta_i - \theta_f)$$



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Scattering from Surface Structures

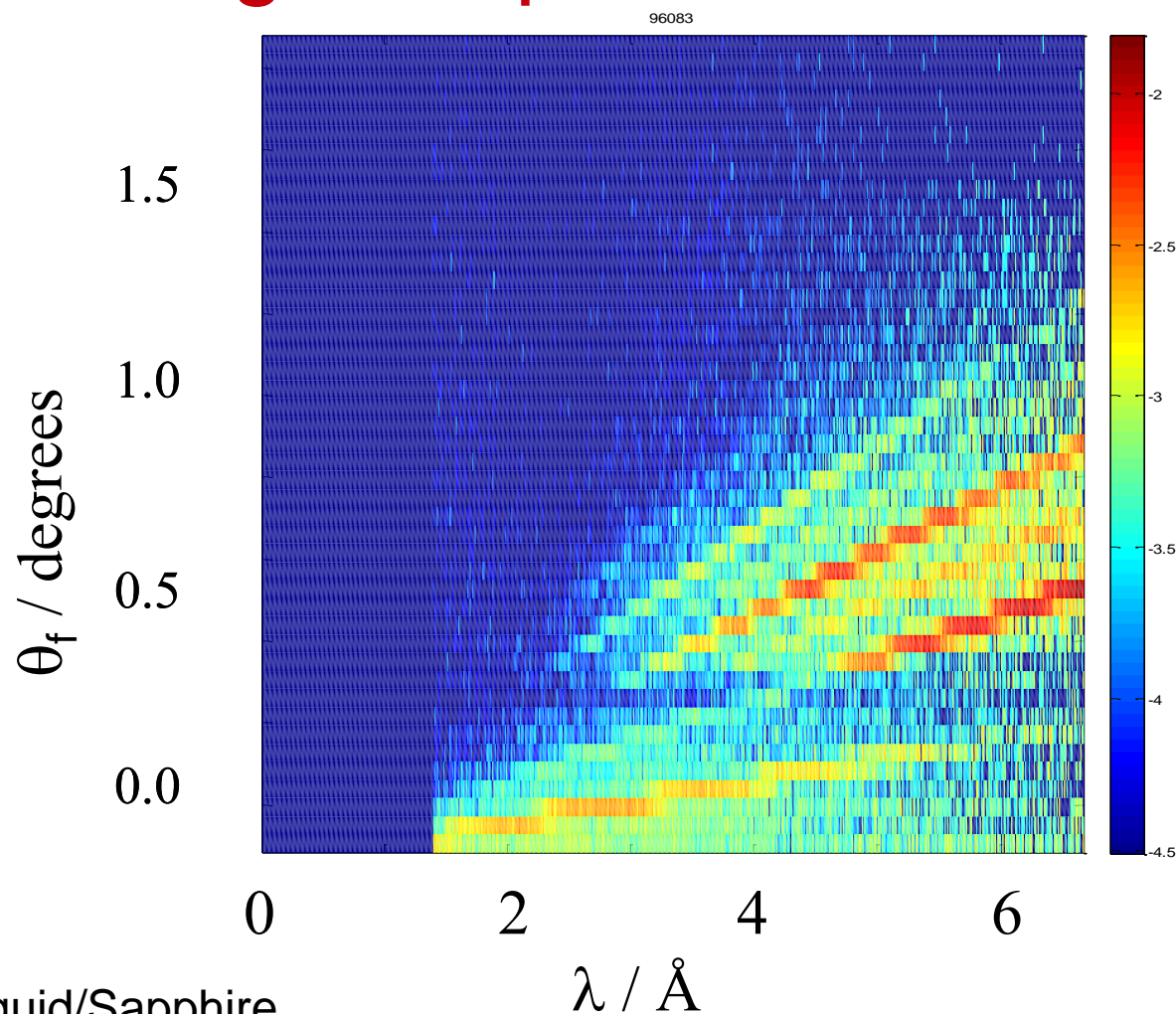


Peter Müller-Buschbaum 'GISAXS and GISANS as metrology technique for understanding the 3D morphology of block copolymer thin films' *European Polymer Journal* **81**, (2016), 470-493.



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Strong Off-specular Scattering



PS latex in D₂O Liquid/Sapphire

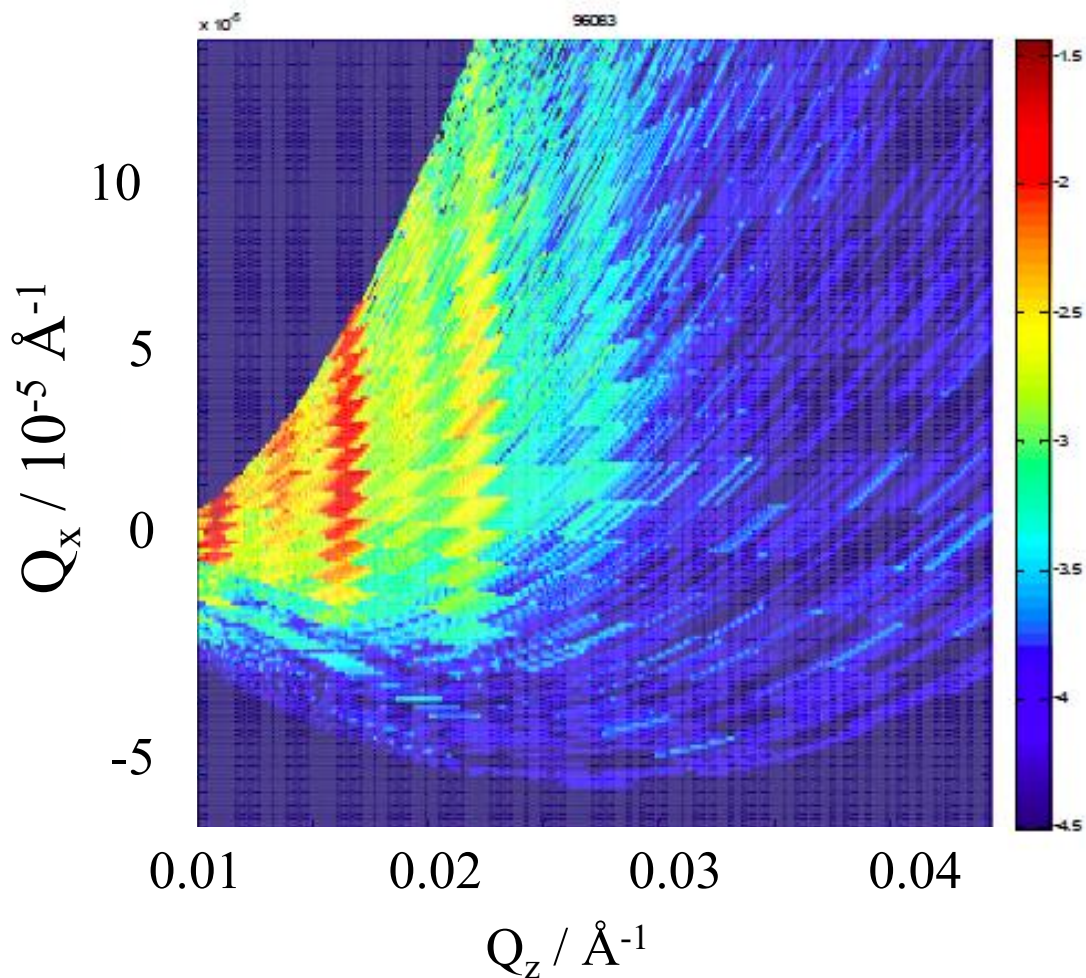
10% vol. dispersion, Radius ~ 350 Å. Sapphire substrate, $\theta_i = 0.35$ deg



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PS latex in D₂O Liquid/Sapphire

Transform to
map of $Q_z Q_x$



10% vol. dispersion, Radius $\sim 350 \text{ \AA}$, sapphire substrate, $\theta_i = 0.35 \text{ deg}$



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Some Scattering at Interfaces

X-ray scattering – glass

Sinha et al., *Phys. Rev.
B.* **38**, 2297, 1988.

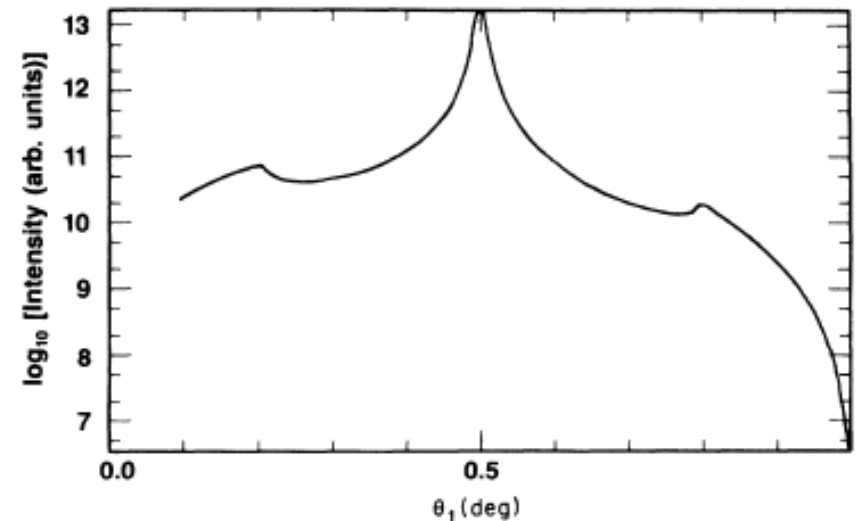
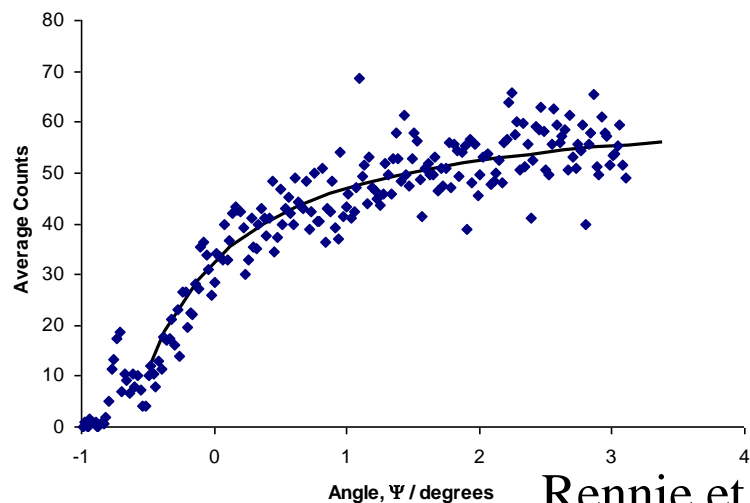
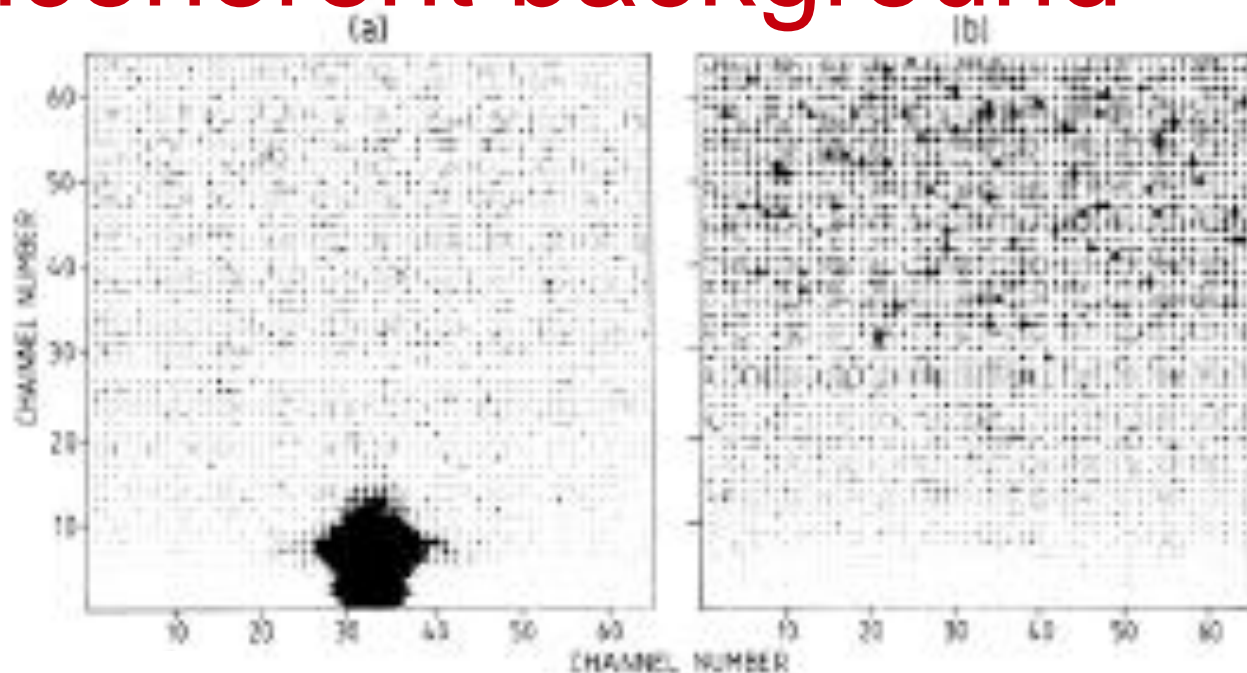


FIG. 6. Calculation of diffuse scattering in the distorted-wave Born approximation for rocking curve where θ_1 and θ_2 are varied such that 2θ is fixed at 1° . The asymmetry is due to the area of the illuminated surface decreasing as θ_1 is increased. The q_y direction has been integrated over. Parameters are $\sigma = 7 \text{ \AA}$, $h = 0.2$, $\xi = 7000 \text{ \AA}$, and the optical constants for Pyrex are given in Sec. V.



Incoherent background



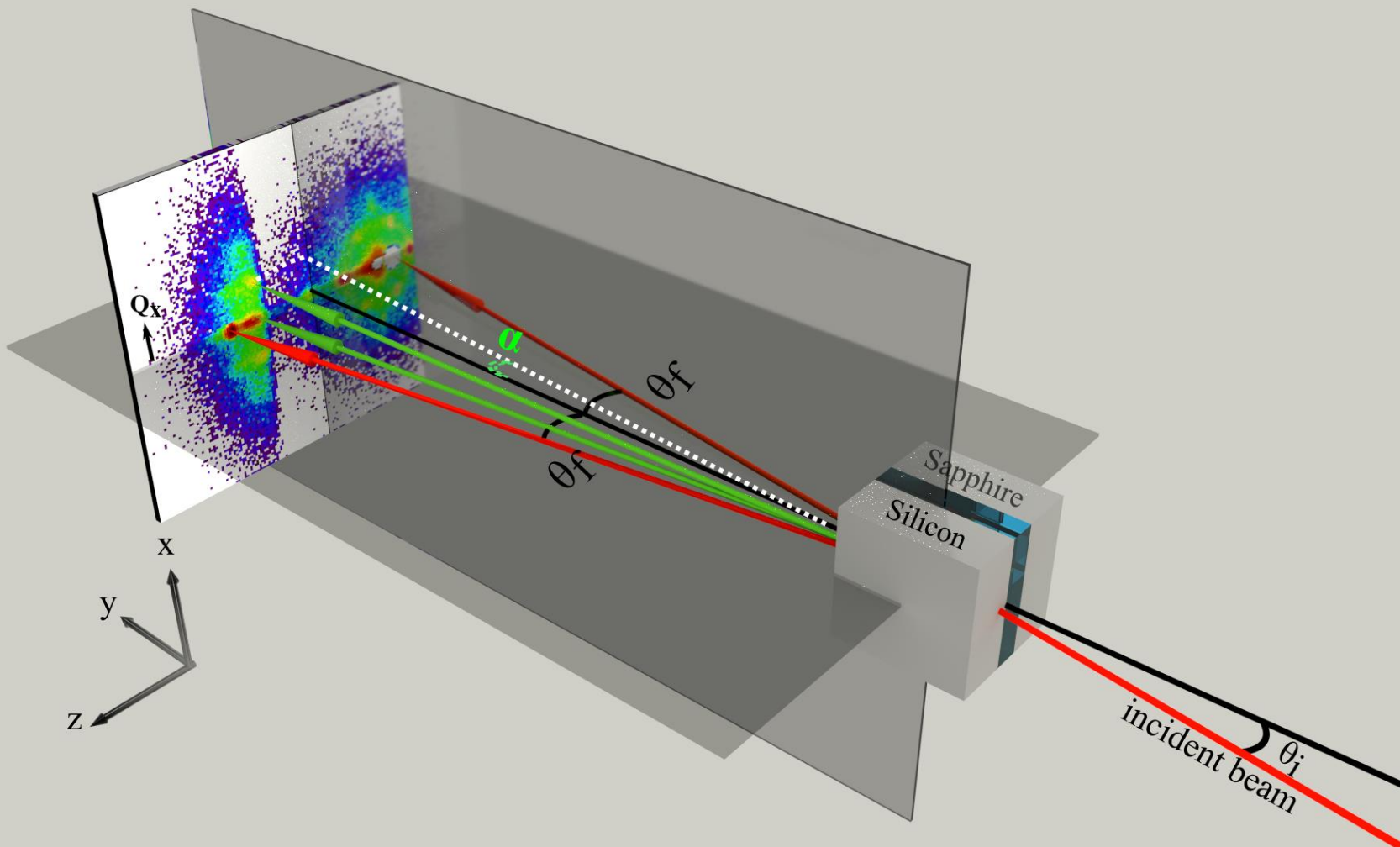
Scattering from D_2O
and from null reflecting water
(8% D_2O)

Rennie et al., *Macromolecules* **22**, (1989), 3466-3475.



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Interfacial structure: GISANS





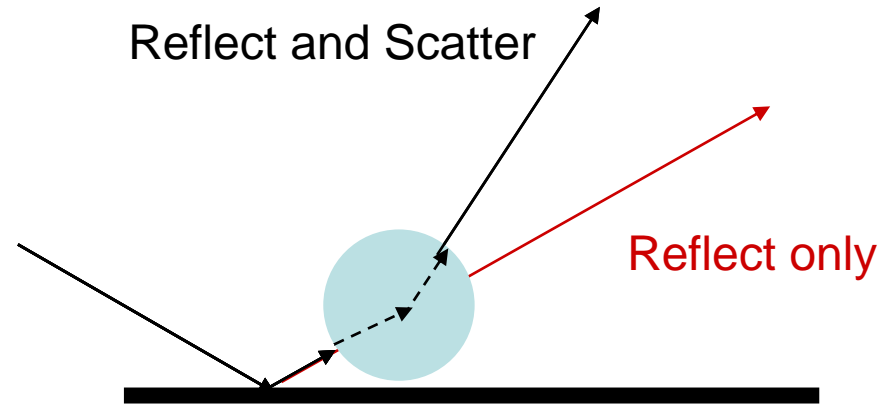
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Calculating Scattering

Distorted Wave Born Approximation (DWBA)

Simply allow for
sequential events e.g.

Reflection then Scattering
Refraction then Scattering
Scattering then Reflection



Reflection followed by weak
scattering.

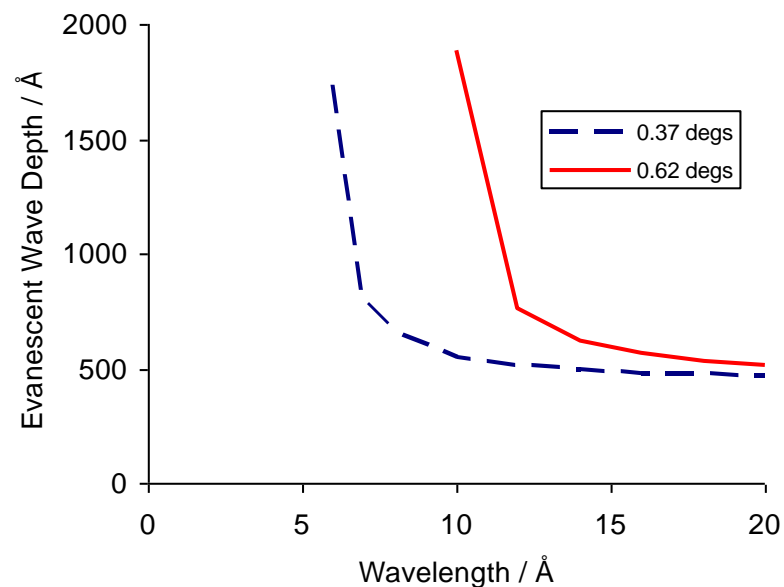
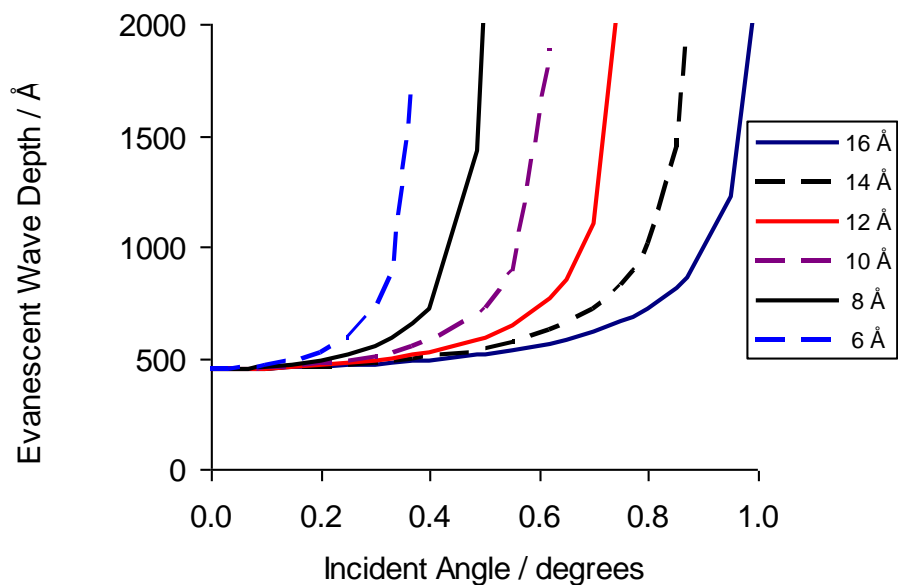
(a) Optical Matrix Calculation

(b) Weak Scattering (Born
approximation)



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How deep is the evanescent wave?

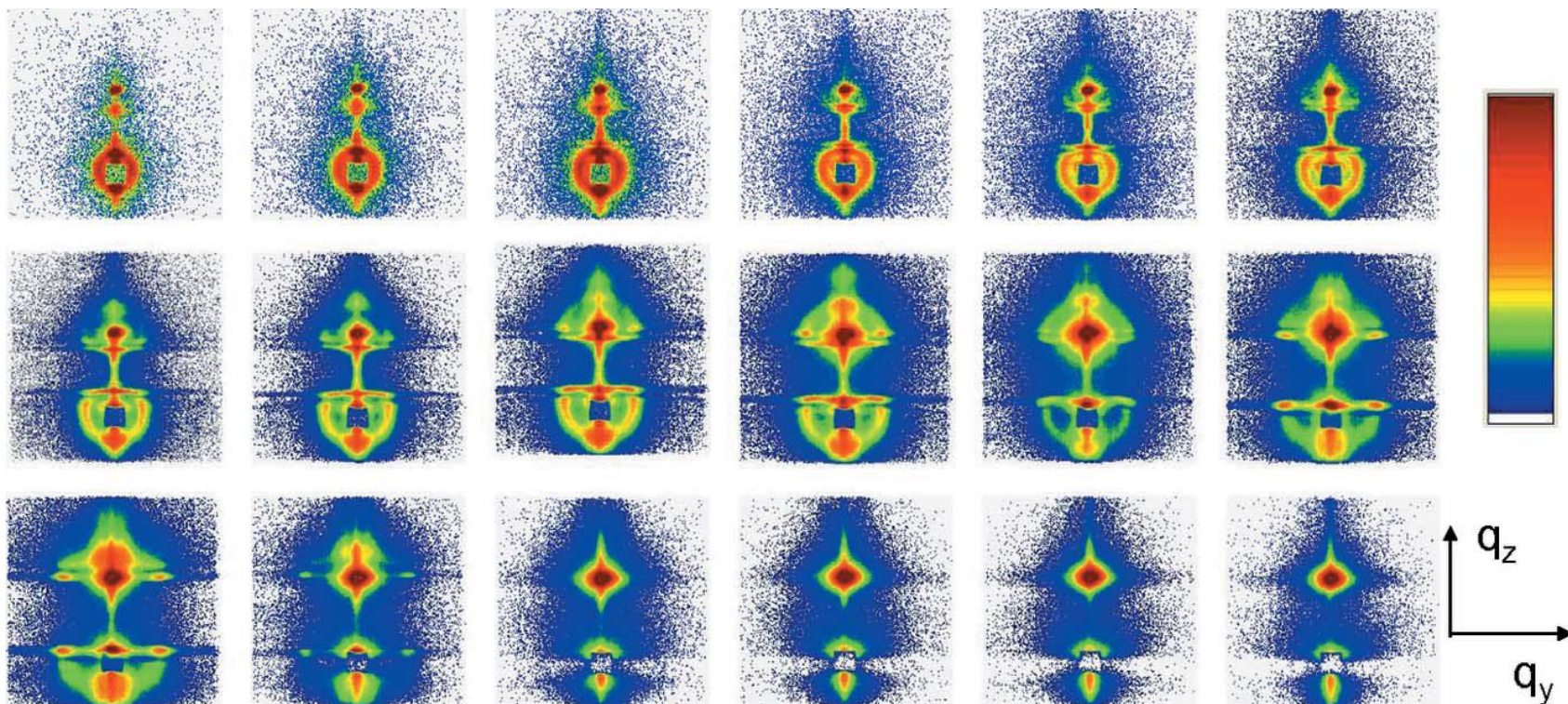


Silicon/D₂O Interface



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Copolymer films



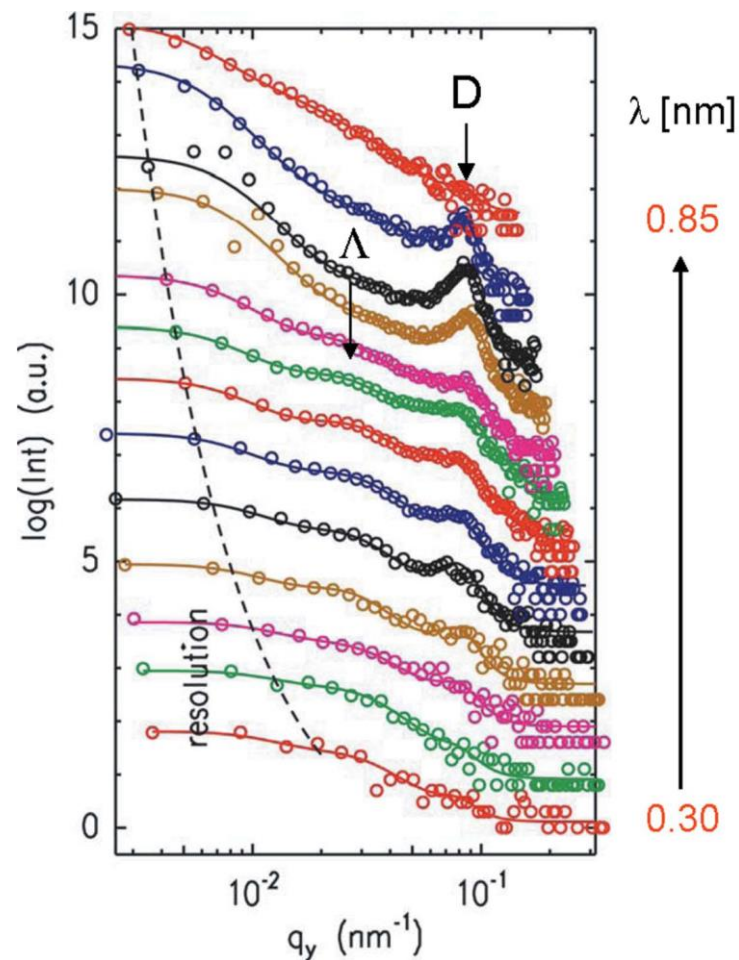
P. Müller Buschbaum *et al.* *J. Appl. Cryst.* **47**, (2014), 1228–1237



Changes with Depth

Horizontal cuts

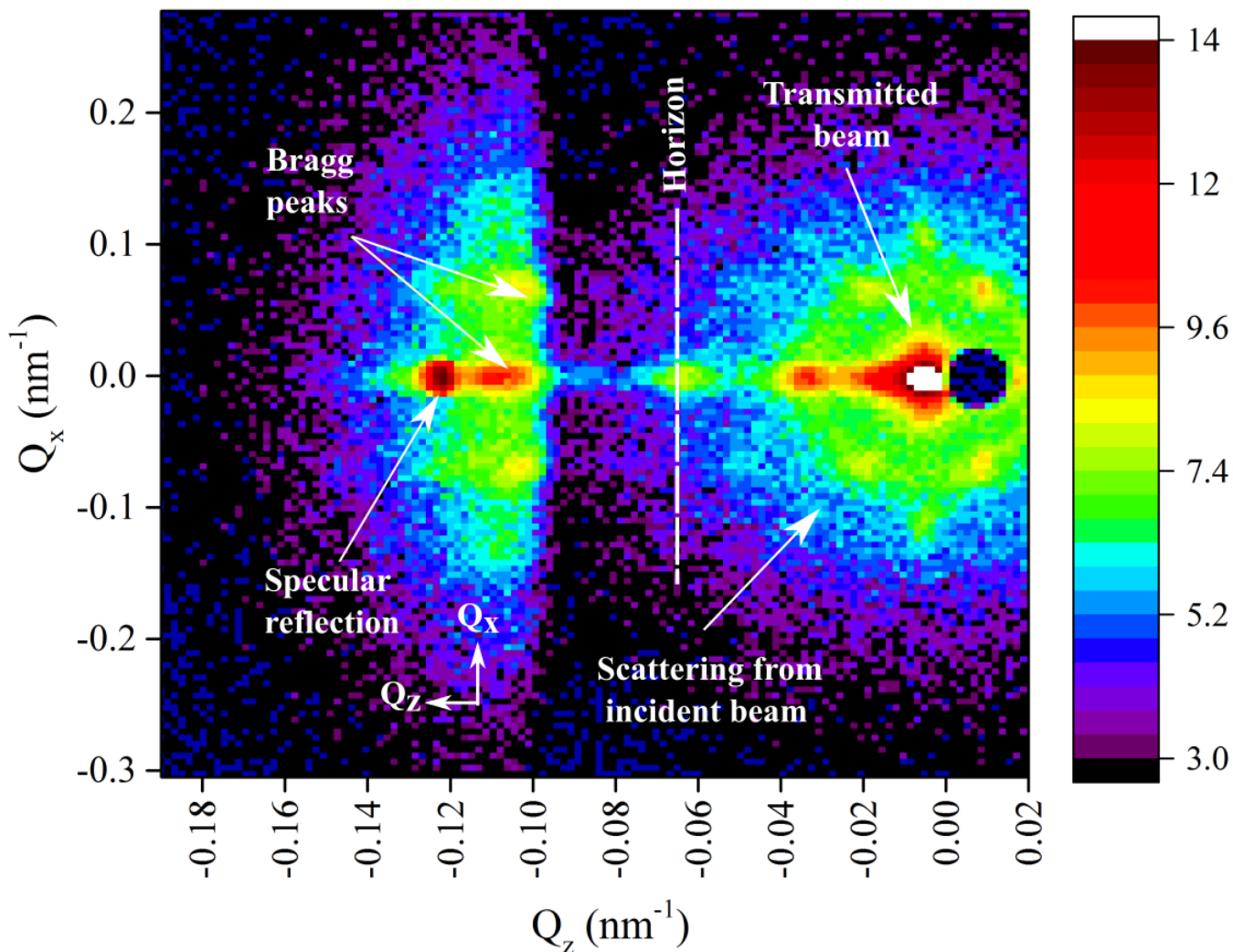
- Used wavelength to probe different depths
- Longer wavelength looks nearer the surface





Diffraction from Surface Layers

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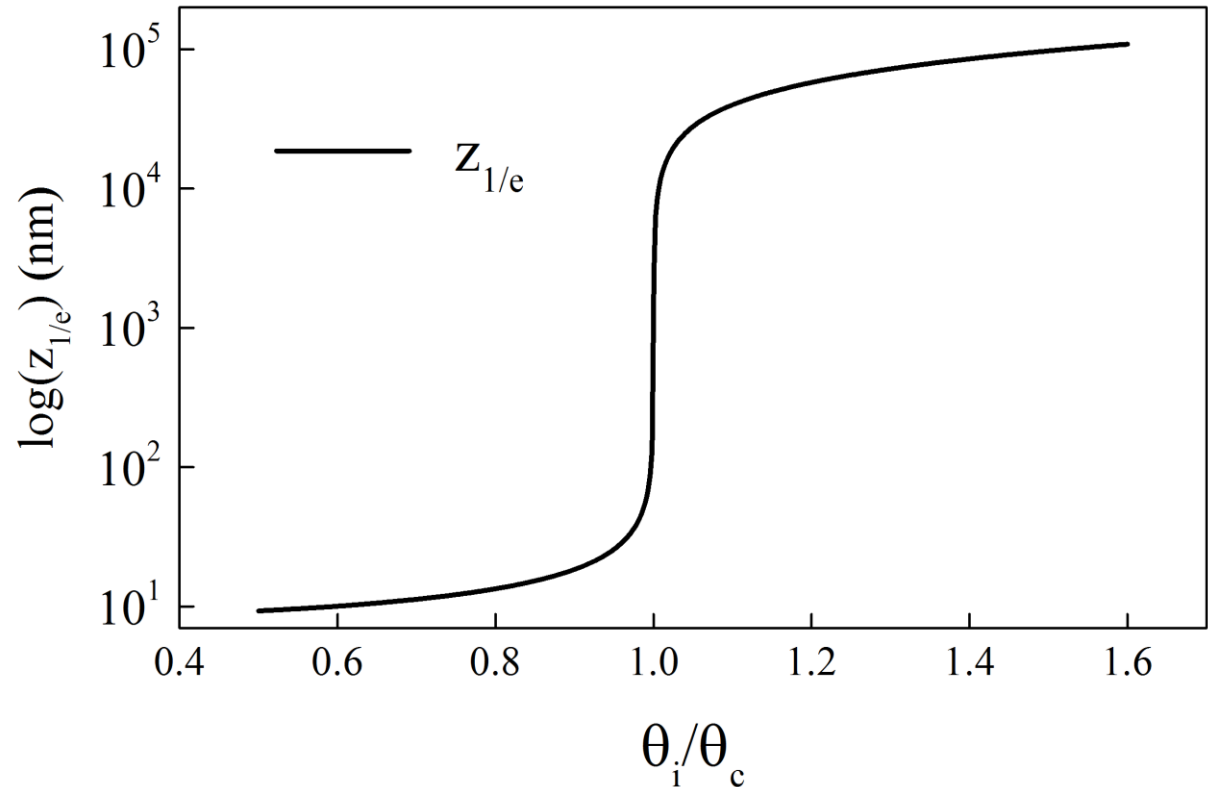
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Penetration depth

$$z_{1/e} = \sqrt{2}\lambda / 4\pi \left[\sqrt{(\theta_i^2 - \theta_c^2)^2 + \left(\frac{\lambda}{2\pi}\mu\right)^2} - (\theta_i^2 - \theta_c^2) \right]^{1/2}$$

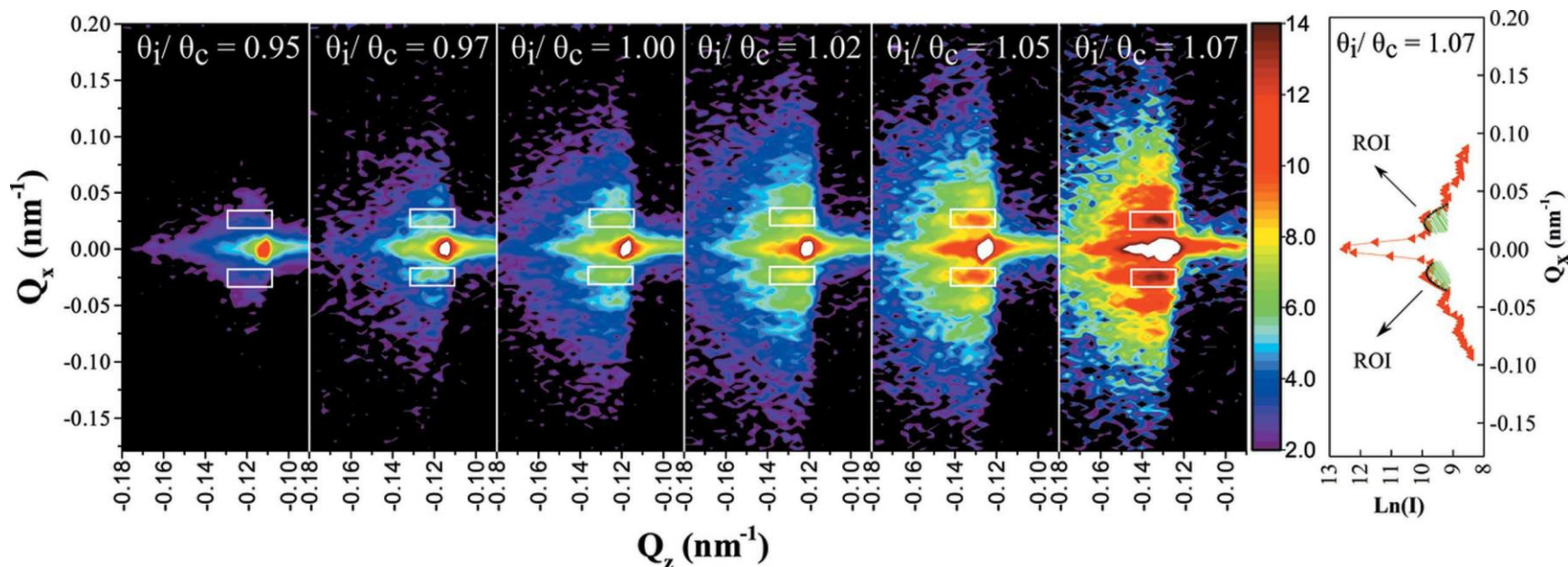
A depth sensitive
technique:

Wavelength
Incident angle



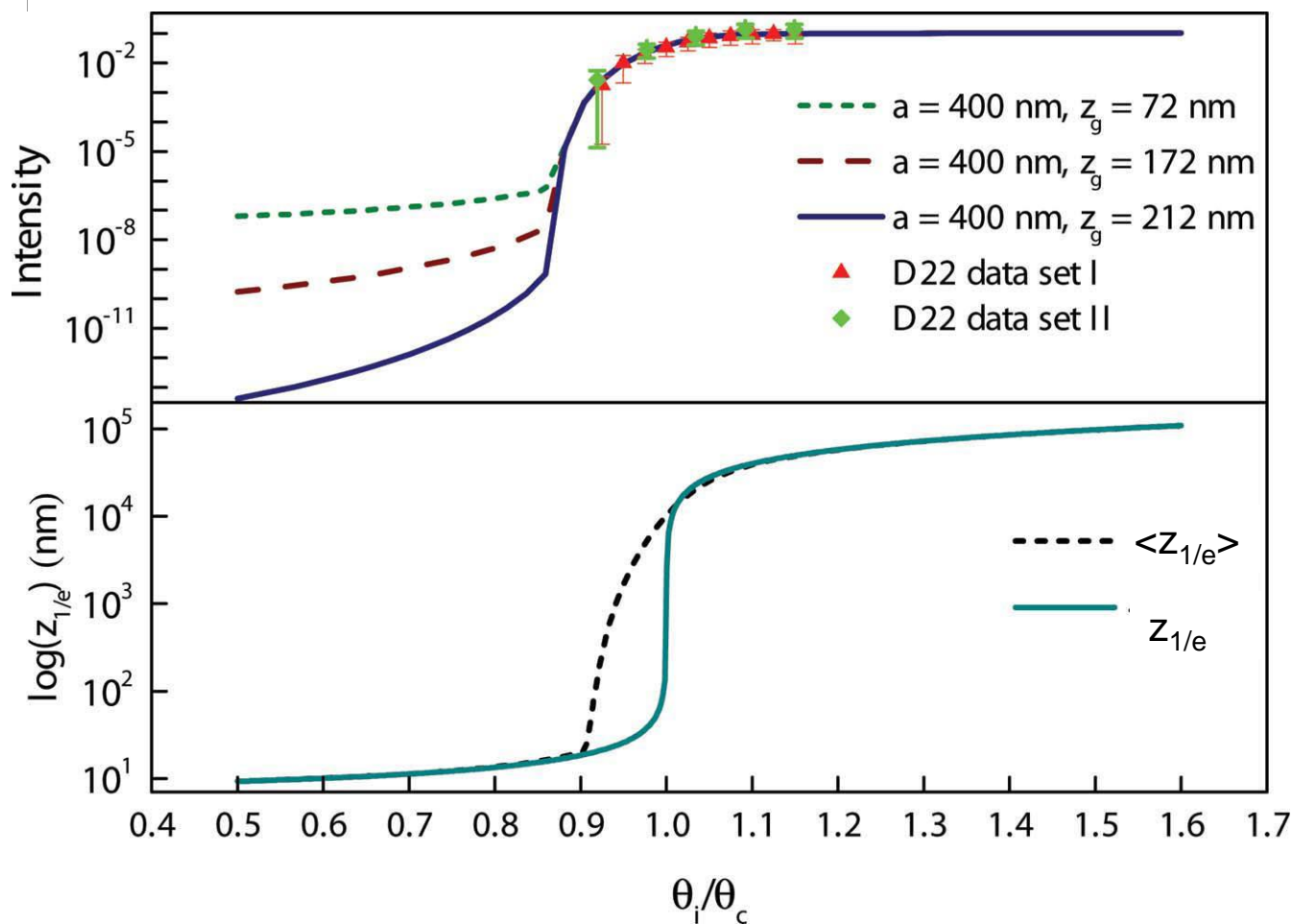


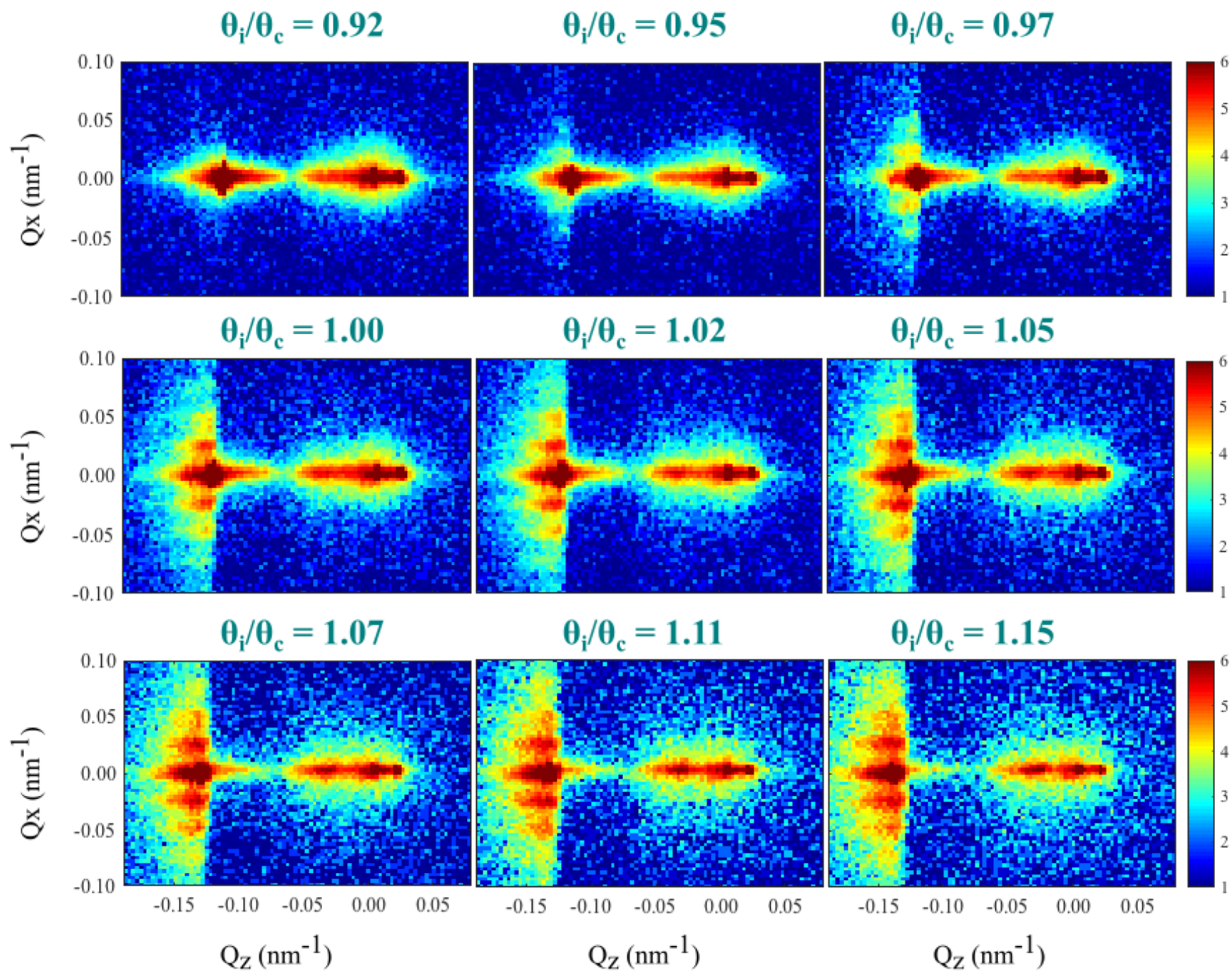
Data at different angles





Data at different angles

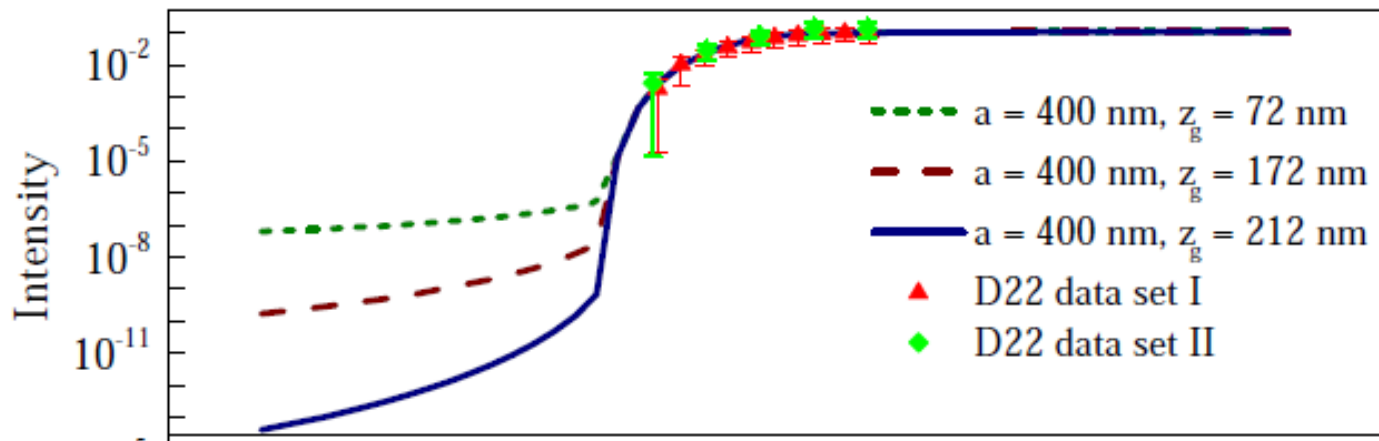






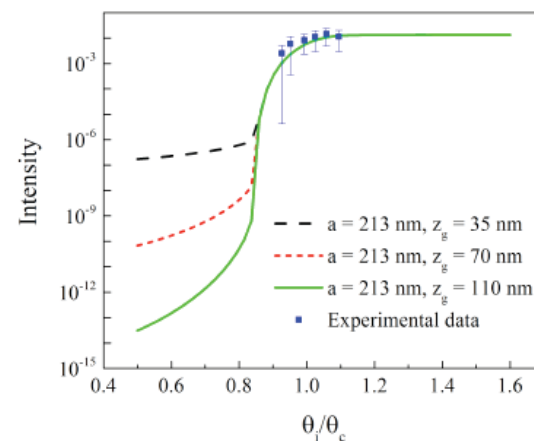
Calculations & Intensity Data

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D22 - ILL

QCM-D data: structure forms with a separation from the interface [Hellsing et al. 2017, *manuscript*]



NG3 SANS - NCNR



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Scattering at Interfaces

- Off-specular scattering
- Near Surface SANS
- GISANS

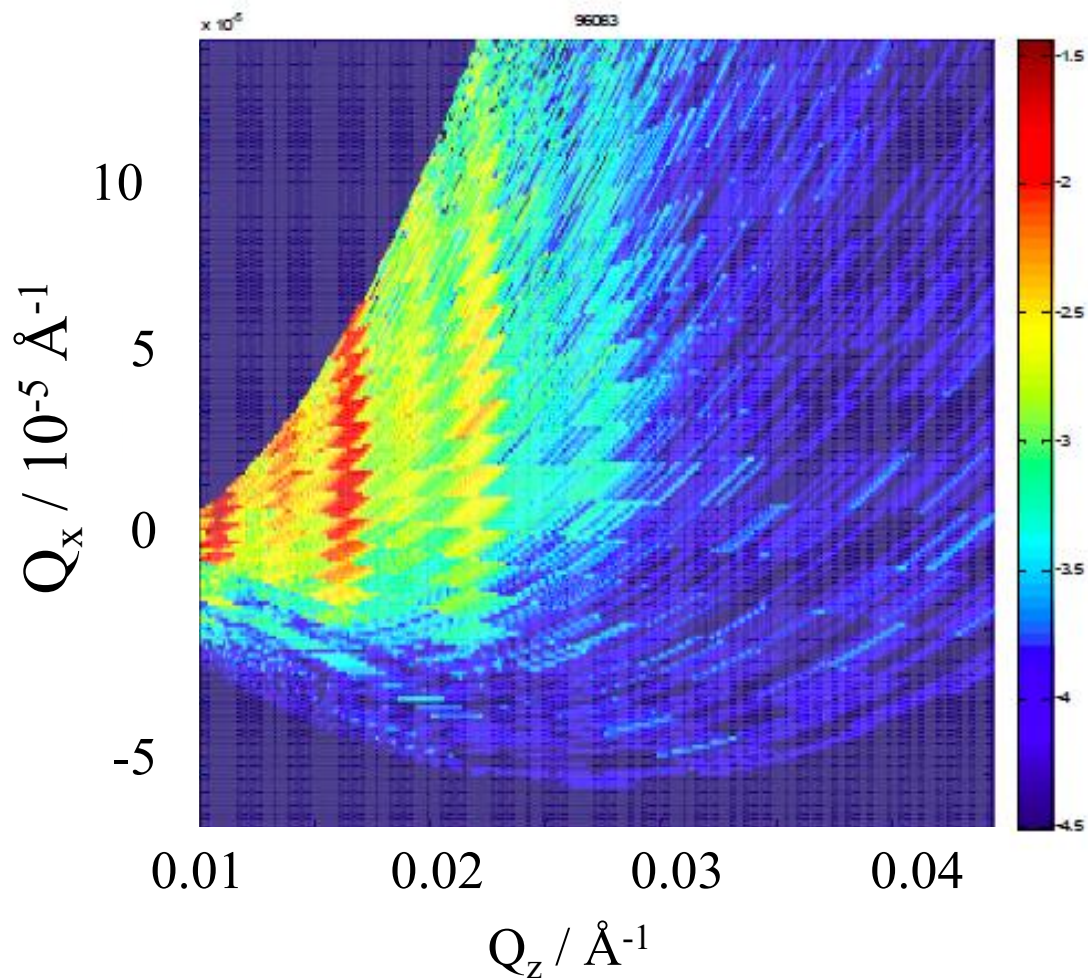
What is the difference?



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PS latex in D₂O Liquid/Sapphire

Transform to
map of $Q_z Q_x$



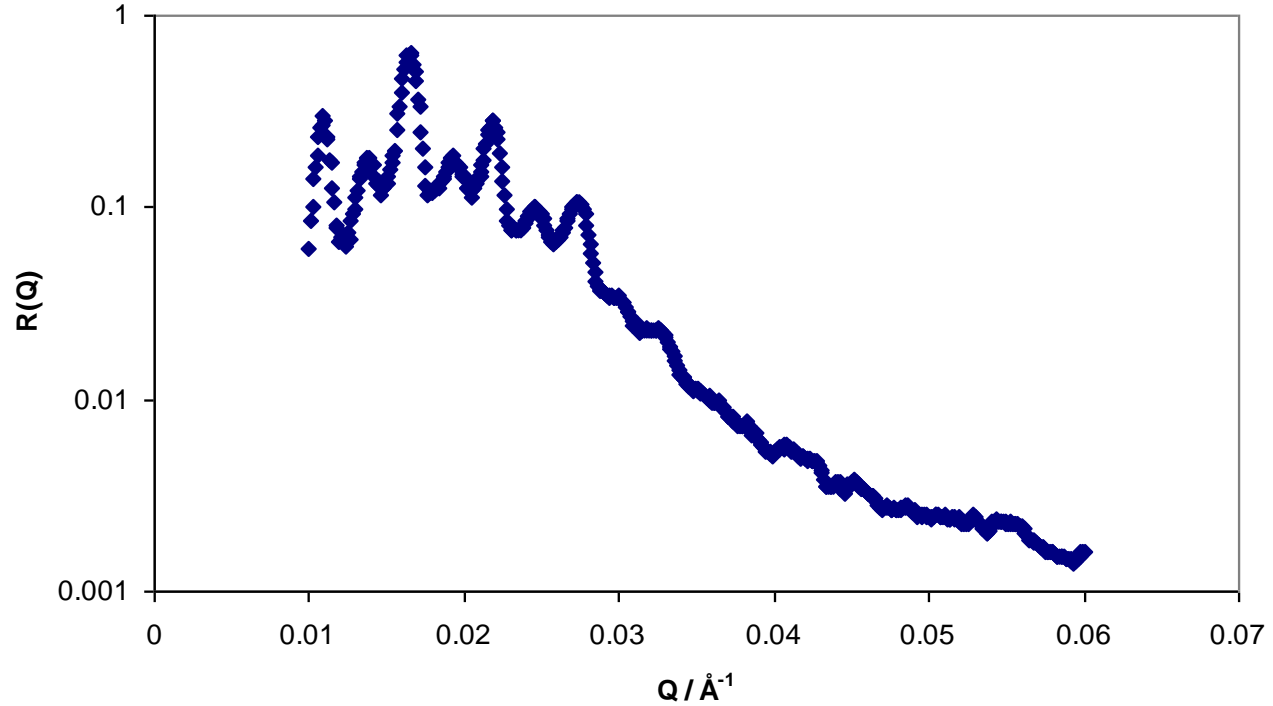
10% vol. dispersion, Radius $\sim 350 \text{ \AA}$, sapphire substrate, $\theta_i = 0.35 \text{ deg}$



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PS latex in D₂O – sapphire surface

Sum along Q_x



10% vol dispersion, 0.35



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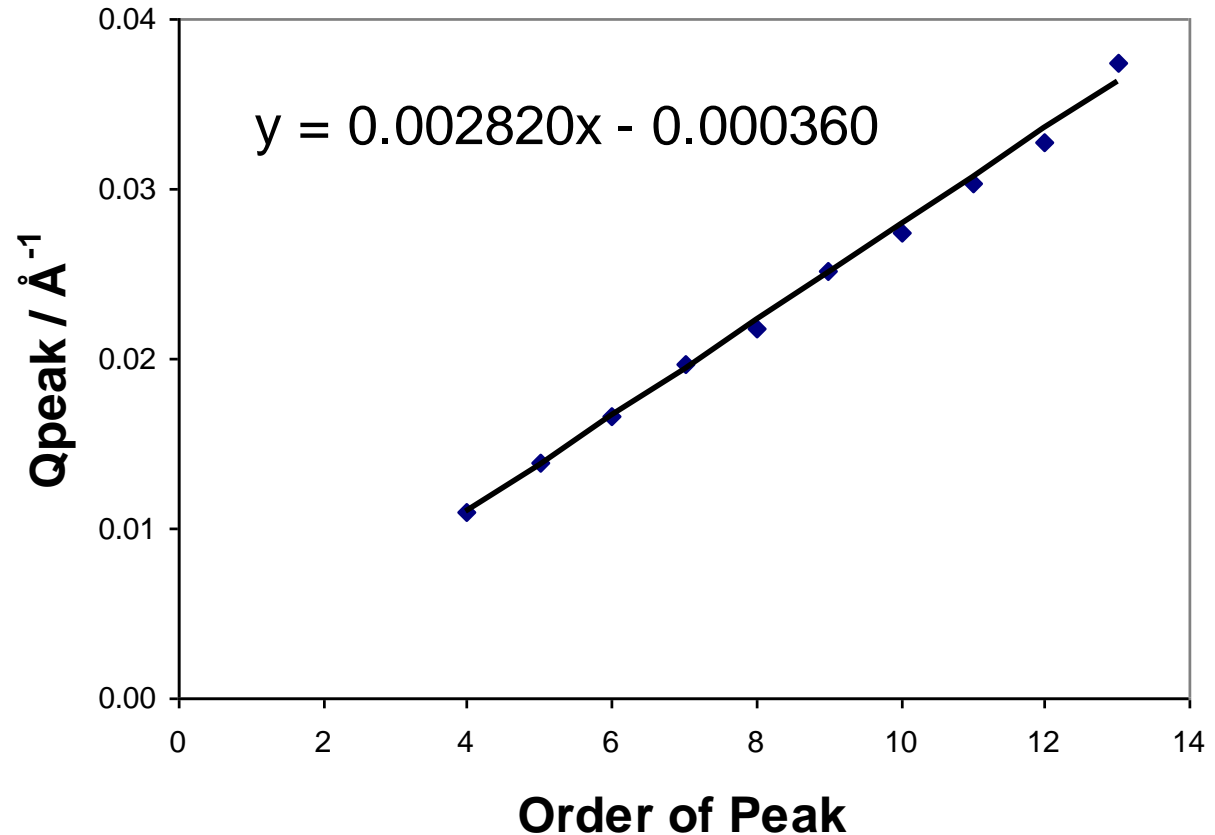
PS latex in D₂O – sapphire surface

Assign Bragg
peaks (index)

$$Q_1 = 0.00282 \text{ \AA}^{-1}$$

$$d = 2230 \text{ \AA}$$

3 first peaks
outside range



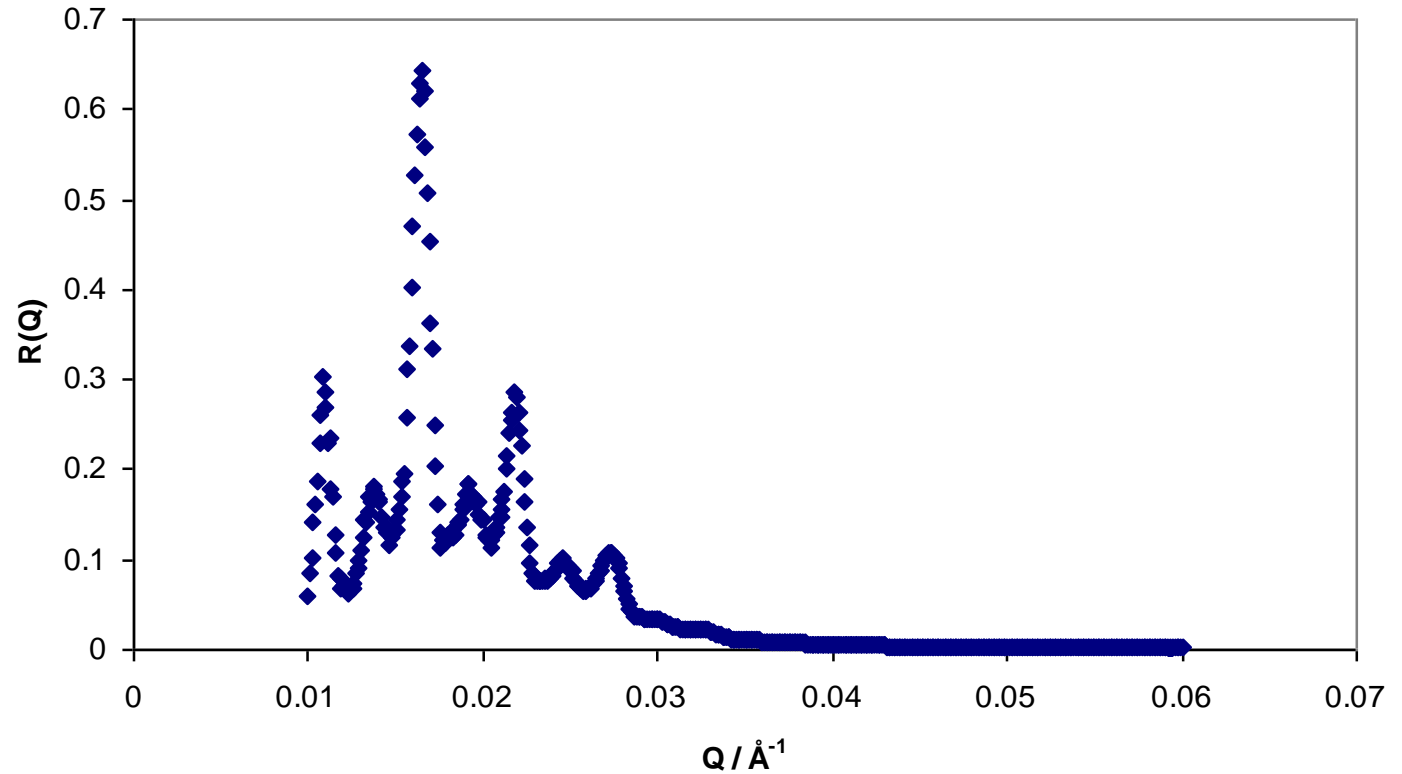
10% vol dispersion, 0.35, 0.8 and 1.5 deg



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PS latex in D₂O – sapphire surface

Sum along Q_x

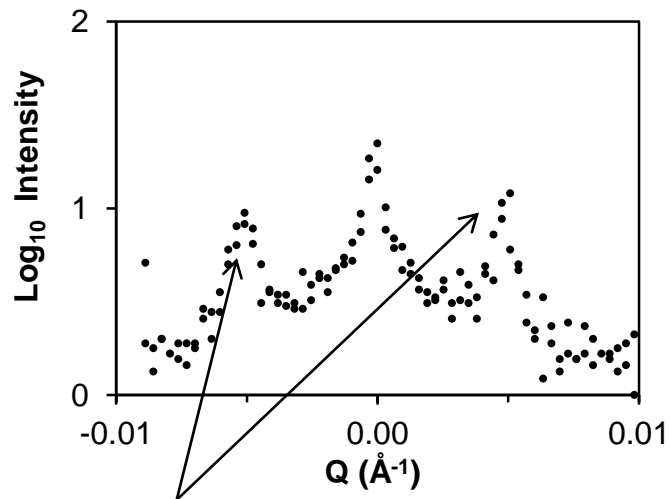
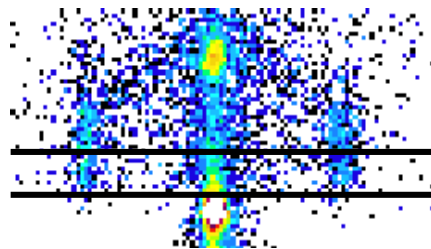


10% vol dispersion, 0.35



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Compare Qx and Qz



M. S. Hellsing, et al. *Applied Physics Letters*, **100**, (2012), 221601.